# Multi-Objective Optimization Algorithm Based on Game Theory and Its Application in Scheduling of Real-Time Tasks

**Lin Chen[1,2*], Xianjia Wang[1,3]**

[1]*Institute of Systems Engineering, Wuhan University, Wuhan, 430072, China*

[2] *School of Information Science and Engineering, Wuhan University of Science and Technology, Wuhan, 430081, China*

[3]*School of Economics and Management, Wuhan University, Wuhan, 430072, China*

*Received 1 November 2014, www.cmnt.lv*

**Abstract**

Biological through survival of the fittest competition, to optimize today's nature. Optimization is an effective way to draw inspiration from the natural evolution of human beings to solve its difficulties; it provides a general framework for solving complex optimization problems. And compared with the game theory demands completely rational assumption, game is on the premise of limited rational optimization algorithm, the game parties by repeating the game in the process of learning, imitation, competition, finding a good strategy, improve their own interests, finally the structure is achieve a dynamic balance on the basis of the optimization algorithm of game theory, this paper studied the production scheduling problem based on evolutionary game. First outlined the game related theory, optimization algorithm and multi-objective optimization algorithm for a class of real-time task scheduling problem, the corresponding evolutionary game model is established, using heuristic genetic algorithm to obtain the corresponding equilibrium solution, using multi-objective optimization algorithm is demonstrated by the simulation results of game theory to solve production scheduling problem is a good idea and has a broad development prospects.

*Keywords:* Game theory; Multi-objective optimization algorithms; Task scheduling; Game model.

## 1 Introduction

In classical game theory, requires that participants be completely "rational", that is, each participant through the spectacular and the best policies to maximize the benefits, but participants are often not completely "rational" but limited "rational". Due to limited knowledge of the participants, limited reasoning ability, limited availability of information collecting and processing capabilities, participants' decision-making behavior is not always maximize, the results of its decision-making influenced by the participant's environment, experiences, habits and other behavior under similar circumstances and other factors [1]. Under limited "rational" conditions, because the participant cannot obtain all of the information that needed, or gained all of the information needed but because of the limited analysis capacity and is unable to conclude that the best decision.

In recent years, with limited "rational" assuming that the optimization algorithm based on game theory has been more and more attention, evolutionary game theory, combined with the classical game theory and evolutionary theory research results, take participants in the group limited "rational" as the research object, using the dynamic analysis method for the inclusion of the various factors of affecting the behavior of participants in the model, and by the viewpoint of system theory to examine the evolution trend of the crowd behavior [2].

## 2 Basic Contents of Game Theory

Game theory emphasizes the system to achieve a balanced dynamic adjustment process, the equilibrium of the system is considered to reach equilibrium function of the process, also said equilibrium depends on the achieve equilibrium path.

A. GAME MODEL

The basic Model of game theory can be divided by the number of group examined single group Model (Monomorphic Population Model) and Model with multiple groups (Polymorphic Populations Model). Single group model direct from ecology research, due to the biological behavior is determined by its genes only, thus it can be species in the ecological environment program for a specific pure strategy [3]. After such treatment, the entire group is equivalent to a select different pure strategy (pure strategy set is equivalent to the number of groups in the population) of the individual. In 1980, Selten [4] multiple population model is put forward, and he by introducing a role with limited in traditional single population ecological evolution model, can distinguish different small groups from large groups, groups of randomly selected individuals in the true sense between two pairs repeat, anonymous asymmetric game, Selten prove [5] the "in multiple population evolutionary

* ***Corresponding author's*** e-mail: clinchenlc@163.com

stable equilibrium in the game are strict Nash equilibrium".

According to the impact factor of the group in the course of evolution is certainty or random, optimization algorithm game model can be divided into deterministic and stochastic dynamic model. Deterministic models is compared commonly simple and can be well described the evolution trend of the system. Stochastic model need to consider many random factors influence on dynamic system, generally is more complex, but this kind of model can more accurately describe the behavior of the system.

### B. APPLICATION OF GAME THEORY

With the mature of evolutionary game theory, the multi-objective optimization algorithm of game theory in the network, electric power market bidding, workshop management, and other fields has been a certain application. Zhi Yang and Shao-xian Ma studied electric power market bidding behavior of power suppliers in stable equilibrium problems, the method using evolutionary game theory under the condition of limited rational to set up the evolution model of power generators bidding, analyzed the earnings on its bidding behavior, analysis shows that only by making a reasonable market mechanism, make the power generation enterprise profit distribution justice, that we can make our country electric power market as soon as possible to achieve a virtuous cycle of state. David and Manoj studied workshop management method of optimization algorithm of game theory, proposes a distributed real-time workshop management system, using evolutionary game method to solve machine scheduling problem, regard the processing tasks as a game player, regard machine as a proposed policy set, constructs the processing task participants choose different strategies machine revenue function, through the comparison of four kinds of commonly used machine scheduling method, verified the effectiveness of optimization algorithm game in large-scale machine scheduling.

## 3 Real-Time Task Scheduling Problem

In practice, due to the equipment processing capacity limit, product technical requirements or to the special properties of the processing tasks these reasons, it will increase the constraint conditions of real-time task scheduling, resulting in a more complex problems, real-time task scheduling problem is a special kind of scheduling problem. If the advanced equipment in machining task is completed, the subsequent equipment are not ready or are in the machining of other tasks, in order to ensure that no waiting for the constraint conditions, to be without waiting for the beginning of the first equipment operating time for appropriate delay, thus the whole machining process on every equipment is continuous.

Real-time task scheduling problem based on fuzzy lead time can be described as: n processing tasks on m different machine processing, a processing task has m step, n processing tasks in the m machine to process in the same order, a processing task at a time can only be processed on one machine, a machine can only at a certain moment in a machining task, there is no waiting time between two of the same processing task process. The machining time of tasks on each machine is given, the relation between customer satisfaction and the delivery date is also given. Requirement of machining sequence of tasks on the machine balance of the interests of its clients.

## 4 Multi-Objective Optimization of Real-Time Task Scheduling Game Model and Its Solution

### A. MODEL DEFINITION

Generally, the game model consists of three main parts: there is a mutual influence of multiple players, each player's strategy set, each players benefit after adopting policy of their respective functions. Based on optimization of game theory, mapping the mathematical model based on real-time task scheduling to optimize game model [7]. Defined as follows: $G=\{N, S_i, U_i\}$.

In the type: N is players, corresponding to each customer's processing tasks, namely the number for processing tasks, is players policy set, corresponding to each customer order processing, players can choose any processing ordinal position, but all the players of the processing order position cannot be repeated. A is player's return functions, corresponding to each customer satisfaction of processing task completion time, according to the idea of evolution game, all customer hope to compete the best processing position to gain the biggest satisfaction, if a customer satisfaction under minimum threshold, the customer will change their strategy, seek better processing position.

### B. MODEL SOLUTION

There are many ways to seek optimization algorithm of game equilibrium, literature [6] overview optimization algorithm game decision-making mechanism, given methods based on Replicator dynamics, stochastic processes, intelligent, neural networks, reinforcement learning. Multi-objective optimization for the proposed real-time task scheduling model of evolutionary game scheduling problem, this section proposes a heuristic genetic algorithm, using genetic algorithm to find the better processing sequence, after the crossover and mutation operators of genetic algorithm using competitive strategy to the machining task sequence of fine-tuning and start time adjustment

(1) Genetic algorithm design

Chromosome by various customers processing task sequence encoding, a scheduling scheme is a processing task arrangement, arrangement of the serial number of

each location corresponding to each customer processing tasks. Such as a chromosome is $\{j_1, j_2, \cdots j_n\}$, shows that processing order of task is $j_1, j_2, \cdots j_n$ .The objective function [8] is:

$$\max f_{sum} = \max \sum_{i=1}^{n} w_{j_1} \mu_{j_1}(C_{j_1、m})$$

Among them $w_{j_1}$ is weight coefficient, $w_{j_1} \in [0,1]$, shows customer attention. $\mu_{j_1}(C_{j_1、m})$ is processing task j fuzzy time of delivery's customer satisfaction membership function. The algorithm is optimized by selection, crossover and mutation operation of three kinds of evolutionary operators.

(2) Heuristic design of competition policy

Because of the real-time tasks of each different customer delivery time, better individuals who are obtained by optimization of individual customer satisfaction may be lower. Based on the demand of customer competition, low degree of satisfaction customer must rob a better processing position, so that the chromosomes sorting change.

And optimization of the default processing task is sequential processing in turn. After start the processing, if the first process is free, the back processing tasks o be started at once in the first process. When the processing tasks completed in advance leads to larger customer satisfaction decline, only by adjusting processing order may not be able to meet the requirements (because the machining task adjust order is bound to affect the other tasks, and other processing task if satisfaction decline it will rob back to its original position in the competition). When this happens, in order to satisfy the customer delivery time, needs to be inserted into the waiting time before machining task start.

For better individuals, which are obtained by optimization, according to each customer satisfaction threshold, if the customer satisfaction is, do not make an adjustment, if the customer satisfaction is, according to the following heuristic competition strategy to handle:

For processing tasks, if the machining task completed delayed, do not deal with. For processing tasks (I = 2, 3,..., n), if the processing task completed delayed, then machining task switching positions, if after exchange, comprehensive customer satisfaction increasing, confirm the exchange success, if confirm exchange conversely failure, restore the original order. For processing tasks (I = 2, 3,... n - 1), if the processing task was finished ahead of schedule, and machining task switching positions, if after exchange comprehensive customer satisfaction increasing, then confirm exchange success, confirm exchange conversely failure, restore the original order. In order to satisfy the customer delivery time, inserted into the waiting time before machining task start, and. For processing tasks, if the processing task was finished ahead of schedule, directly inserted into the waiting time before machining task start.

The choice of customer satisfaction threshold should be appropriate, if small leads to smaller comprehensive customer satisfaction. If choose a few bigger, although can make bigger comprehensive satisfaction, but always can increase the task completion time, excessive take machine time and cause machine resources waste may also affect own benefits[9].

After insert waiting time, real-time task scheduling model formula is:

q=1,2, ..., m, i=2,3, ..., n

## 5 Simulation Design

Use five real-time tasks, 4 machine cases. Each real-time task completion time and fuzzy lead time value as shown in TABLE 1, 2

TABLE 1 The real time task completion time

| Real time task | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| machine1time | 31 | 19 | 23 | 13 | 33 |
| machine2time | 41 | 55 | 42 | 22 | 5 |
| machine3time | 25 | 3 | 27 | 14 | 57 |
| machine4time | 30 | 34 | 6 | 13 | 19 |

TABLE 2: Real-time fuzzy delivery dates

| Real time task | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| Delivery time $e_j^i$ | 200 | 100 | 190 | 40 | 130 |
| Delivery time $e_j^n$ | 220 | 130 | 220 | 60 | 160 |
| Delivery time $d_j^i$ | 240 | 150 | 250 | 80 | 180 |
| Delivery time $d_j^n$ | 270 | 180 | 270 | 100 | 210 |

To optimize the initial parameter values as follows: population size is 20, cross coefficient is 0.8, the variation coefficient was 0.1, and optimization of algebra is 40. The weight coefficients of various customers is w = (0.5 0.3 0.9 0.6 0.3), customer satisfaction threshold $\xi$ are 0.5.

Game model of optimization algorithm and a heuristic genetic algorithm to solve the scheduling problem, the simulation results (processing task scheduling gantt chart) as shown in Figure 1, in the diagrams, numbers in the box is processing tasks, the lower left corner of the box numbers is processing the start time, the lower right of the box number is processing and end time.

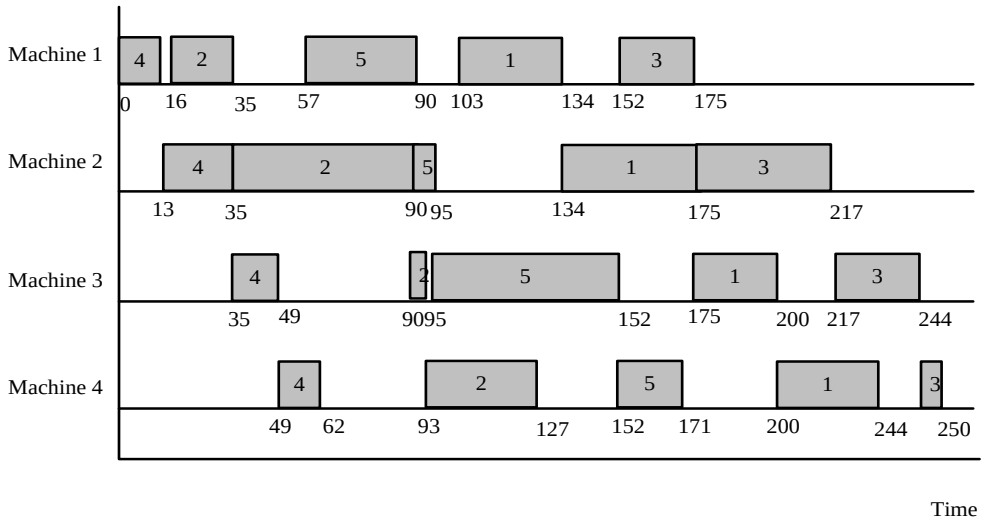


FIGURE 1. Gantt chart of scheduling heuristic genetic algorithm

If only uses the genetic algorithm, heuristic competition strategy operation, the simulation results (processing task gantt chart) as shown in Figure 2.

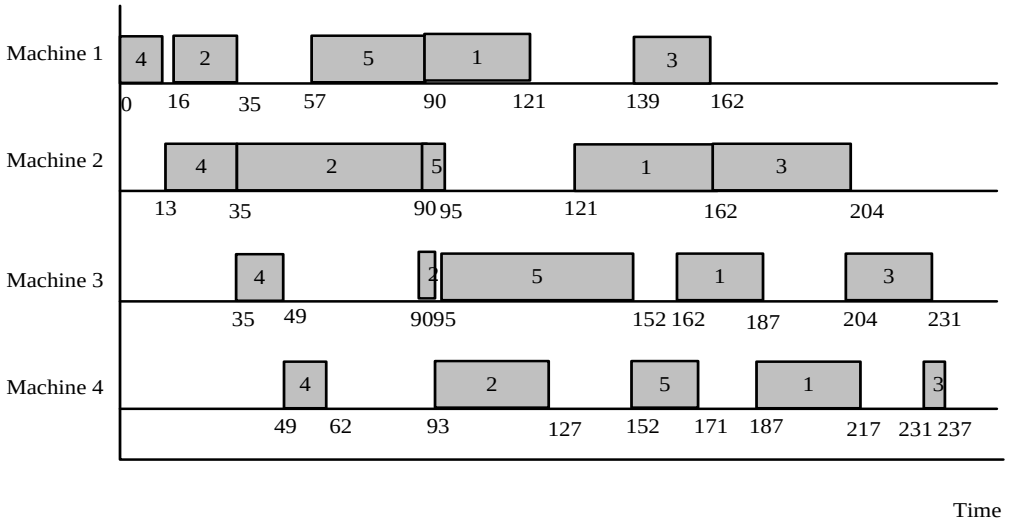


FIGURE 2. Gantt chart of scheduling using genetic algorithm

Can be seen from the Figure 1 and Figure 2, if only uses the genetic algorithm, the total completion time for the processing of all is 237, comprehensive satisfaction is 2.345, the customer satisfaction is (0.35, 0.9, 1, 1, 1) respectively, among them, customer 1's satisfaction is lower than the threshold value; If using heuristic genetic algorithm proposed in this chapter, then all the total completion time for the processing is 250, comprehensive satisfaction is 2.445, the customer satisfaction is (1, 0.9, 0.75, 1, 1) respectively, the processing of the wait time is (13, 0, 0, 0)respectively, in order to make customer 1's satisfaction to be greater than satisfaction threshold.

## 6 Conclusion

Combining with the actual situation of modern production. For a class of real-time task scheduling problem with fuzzy lead time, considering the customer satisfaction, in each of the processing tasks required to meet minimum satisfaction threshold, on the basis of

comprehensive to maximize satisfaction as the goal, for a class of real-time task scheduling problem are described. The multi-objective optimization algorithm of game theory is introduced to build the multi-objective optimization scheduling model, map the scheduling tasks to game theory model, for solving the scheduling model can be converted to optimization of game equilibrium constraint conditions. Fully consider the customer's competition, we design a heuristic genetic algorithm to optimize the game equilibrium, joined the heuristic competition strategy in the course of the algorithm, greatly improving the efficiency of evolution. According to the theory, model and method, in view of a scheduling example simulation, the simulation results show that using multi-objective optimization algorithm game scheduling model to describe the real-time task scheduling problem with fuzzy lead time is reasonable and effective.

## Acknowledgement

National Natural Science Fund 71071119, Major Project of National Natural Science Fund 71231007.

## References

[1] Qi J D, Liu J 2014 The third industrial revolution under the trend of transformation of industrial organization *Research on Financial and Economic* 1 27-33.
[2] Hou J 2013 Impact of technology innovation to enterprise and market structure *Liaoning: Liaoning University*
[3] Zheng S Y 2012 Research progress of cooperative game theory--study on formation mechanism and stability of the League reviewed *Journal of Shanghai Maritime University* 32(4) 53-59
[4] Selten R 1980 A Note on Evolutionarily Stable Strategies in Asymmetric Games Conflicts. *Journal of Theoretical Biology* 84 93-101
[5] Selten R 1991 Evolution, learning, and economic behavior *Games and Economic Behavior* 3(1) 3-24
[6] Hofbauer J, Sigmund K 2003 Evolutionary game dynamics *American Mathematical Society* 40(4) 479-519
[7] Wang W L, Song L, Xu X L, etc 2006 Fuzzy flow-shop scheduling with mixed intermediate storage strategies *Computer integrated manufacturing systems* 12(12) 2067-2073
[8] Liu Y, Zhang S D 2012 Price with fast based on evolutionary game theory under the angle of convergence *Logistics engineering and management* 34(1) 73-74
[9] Tu L H, Qu F Q, Li H H 2011 Research on housing promotion work based on evolutionary game model in China *Journal of engineering management* 25(2) 215-219

## Authors

**Lin Chen, Hubei Province of China**

**Current position, grades: lecturer**
**University studies: Master's degree was earned in major of control theory and control engineering, Huazhong University of Scence and Technology in 2007.**
**Scientific interest: control theory, systems engineering, systems engineering**

**Xianjia Wang, Hubei Province of China**

**Current position, grades: professor, doctoral supervisor**
**University studies: PHD degree was earned in major of hydrology and water resources, Wuhan University in 1992.**
**Scientific interest: game theory, Management decision analysis and investment decisions, Industrial organization theory and incentive theory, water and power market management**