

# The research on thermal expansion and thermal compensation method of Rogowski coils

Hongling Zhang, Bin Hong, Yuan Gui, Xiaoqing Yang\*

Hebei construction engineering college, Zhangjiakou, China

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## Abstract

Rogowski coils are mostly used to measure AC current, pulse current and transient current in power industry. Thermal expansion can change the structural parameter of the coils when the ambient temperature varies, so it affects the measurement precision of Rogowski coils. The mathematical model of the thermal expansion effect on measurement precision of Rogowski coils is proposed in this paper. The ratio voltage error caused by thermal expansion effect is identical with the ratio error of coils mutual inductance, that is obtained by Matlab simulation. The thermal compensation method that can eliminate the thermal expansion effect on Rogowski coils is introduced, that is, a compensation ring is embedding in Rogowski coils former. The mathematical model of thermal compensation is proposed, and it is verified by Matlab simulation. The experimental results show that the ratio voltage error tendency with temperature is the same as the error tendency from theoretical analysis and the limitation of measurement precision relates with coils thermal expansion; It may eliminate the thermal expansion effect on Rogowski coils to embed a compensation ring in Rogowski coils former.

**Keywords:** Rogowski coils, thermal expansion, the ratio error, compensation ring

## 1 Introduction

Rogowski coils are increasingly used in detecting and measuring electrical currents in power industry. It is especially so for very high current applications, due to the ease of implementation in the circuit. Rogowski coils can be divided in to three types:

- flexible core coils,
- rigid core coils,
- printed circuit boards (PCB) based coils [1, 2].

The flexible Rogowski coil type is the most widely used, and may be wound on a rectangular or circular air core [3]. The PCB Rogowski type is based upon two PCBs wound in opposite direction located next to each other, and is called a mirrored printed circuit board (MPCB) [4, 5]. Though these studies have been done, fabricating a Rogowski coil that has measurement precision within 0.1% is still a challenge work. The major source limiting measurement precision is thermal expansion effect on coils.

Factors will affect the measurement performance and to a certain extent, restrict the stability of the Rogowski coil, bandwidth, accuracy and precision [6-10], and restricts the precision thermal effect is one of the main reason why the impact on the coil [11-15]. This paper analyzes the thermal effect of Rogowski coil measuring accuracy, the influence of the mathematical model of voltage relative error of the estimate is obtained; And put forward a method to reduce the influence of thermal effects on its, compensation ring embedded in the Rogowski coil former.

## 2 The thermal expansion effect of Rogowski coil former

### A. The ratio voltage error of Rogowski coil

The ratio voltage error caused by thermal expansion

effect is identical with the ratio error of Rogowski coil mutual inductance. That is

$$\frac{u(t) - u_0(t)}{u_0(t)} = \frac{-\Delta M \frac{di}{dt}}{-M \frac{di}{dt}} = \frac{\Delta M}{M} \quad (1)$$

For Rogowski coil having a rectangular cross-section such as shown a figure (see, Figure 1).

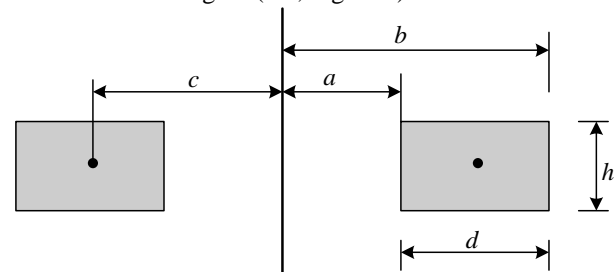


FIGURE 1 Rogowski coil with rectangular cross-section

The mutual inductance of Rogowski coil with rectangular cross-section is given by

$$M = \frac{\mu_0 N h}{2\pi} \ln \frac{b}{a} = \frac{\mu_0 a h}{\lambda} \ln \left( \frac{a+d}{a} \right), \quad (2)$$

where  $M$  is the mutual inductance of coils coupling the measurement conductor;  $\mu_0$  is the permeability of free space,  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ ;  $a$ ,  $d$ ,  $h$  and  $N$  are the inner radii, thickness, height and number of turns of the coil respectively.  $\lambda$  is the diameter of the coil wire.

### B. The thermal expansion effect on the coil.

When the temperature is changed around the coil, the

\*Corresponding author e-mail: ZHL2010\_13@163.com

diameter, thickness and height of the coil will be changed. Sequentially we have

$$\frac{\Delta M}{M} = \frac{\frac{\mu_0 N(h + \Delta h)}{2\pi} \ln\left(1 + \frac{d + \Delta d}{a - \Delta a}\right) - 1}{\frac{\mu_0 N h}{2\pi} \ln\left(1 + \frac{d}{a}\right)} - 1 = \frac{\ln\left[1 + \frac{d(1 + \frac{\Delta d}{d})}{a(1 - \frac{\Delta a}{a})}\right]}{\ln\left(1 + \frac{d}{a}\right)} + \frac{\Delta h}{h} \cdot \frac{\ln\left[1 + \frac{d(1 + \frac{\Delta d}{d})}{a(1 - \frac{\Delta a}{a})}\right]}{\ln\left(1 + \frac{d}{a}\right)} - 1 \quad (3)$$

A succinct expression of Equation (3) is

$$\xi = \frac{\Delta M}{M} = \frac{\ln(1 + \theta \frac{1 + \varepsilon}{1 - \varepsilon})}{\ln(1 + \theta)} (1 + \varepsilon) - 1, \quad (4)$$

where  $\varepsilon$  is the thermal expansion coefficient of the coil winding or former, in that  $\varepsilon = \Delta h/h = \Delta d/d = \Delta a/a$ ,  $\theta$  is a construction factor of Rogowski coil and expressed as  $\theta = d/a$ .

If the coil undergo a temperature changes from  $t_1^\circ\text{C}$  to  $t_2^\circ\text{C}$ , the ratio error of the output voltage is given by

$$\frac{\Delta u}{u} = \xi(t_2 - t_1), \quad (5)$$

where  $\xi$  describes the ratio error of the coil output per unit rise in temperature and its unit is  $^\circ\text{C}^{-1}$ . It can describe the thermal expansion effect on the coil with rectangular cross-section output voltage directly, whether the ratio voltage error is caused by the thermal expansion of winding wire or coil former.

C. Discussion.

In the case of common use, the winding wire is usually copper wires. Its coefficient of thermal effect is about  $16.5 \times 10^{-6}^\circ\text{C}^{-1}$ . The parameters of the coil at temperature  $20^\circ\text{C}$  is shown in (Table 1).

TABLE 1 coils parameters of configuration (at  $20^\circ\text{C}$ )

Sym	Description	Value
M	The mutual inductance of Rogowski coil	0.452 $\mu\text{H}$
$\lambda$	The diameter of winding wire	$2.3 \times 10^{-4}(\text{m})$
N	The number of turns	640
a	The inner radii of Rogowski coil	0.032 5 m
B	The outer radii of Rogowski coil	0.041 1 m
d	The thickness of Rogowski coil	0.008 6 m
H	The height of Rogowski coil	0.015 6 m
$\theta$	A factor of Rogowski coil structure	0.264 6
$\varepsilon$	The thermal expansion coefficient of former	$80 \times 10^{-6}^\circ\text{C}^{-1}$

When the coil undergo a temperature changes from  $-40^\circ\text{C}$  to  $60^\circ\text{C}$ , we calculate the fractional change of the coil,  $\Delta a$ ,  $\Delta h$  and  $\Delta d$  (e.g., Figure 2).

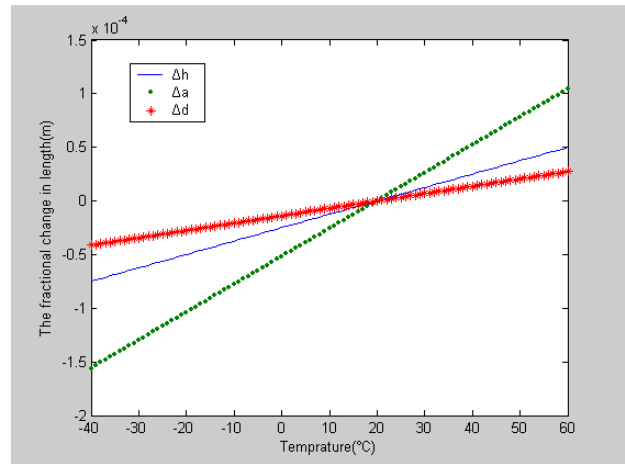


FIGURE 2 The fractional change  $\Delta a$ ,  $\Delta h$  and  $\Delta d$

The resultant data of mutual inductance coil at different temperature are as shown in Figure 3.

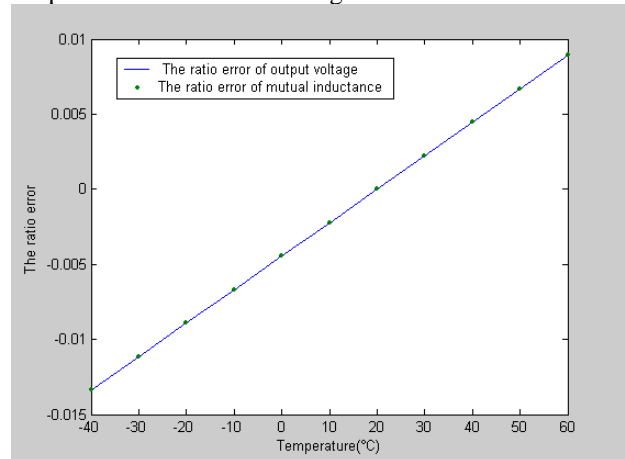


FIGURE 3 The ratio voltage error of coils with rectangular cross section is identical with the ratio error of mutual inductance

Actually, there are two curves in Figure 3. They overlap each other. One is formed according to Equation (3) and the other is drawn based upon the data from Equation (5).

Through the simulation of MATLAB, the simulation diagram as shown in Figure 4.

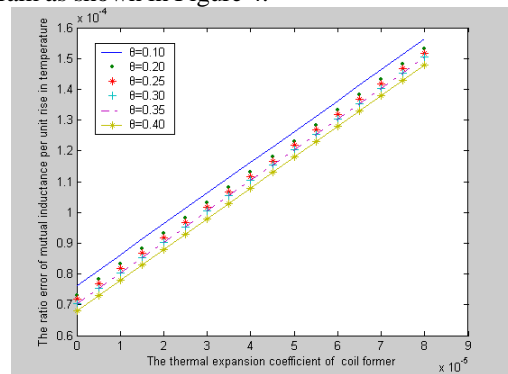


FIGURE 4 The relationship between  $\xi$ ,  $\theta$  and  $\varepsilon$

Figure 4 shows that the ratio error of Rogowski coil mutual inductance is affected by the structural parameters of the coil, that is  $\theta$ , the smaller  $\theta$ , the bigger the influence. If we can reasonably choose the coil structure parameters. when the temperature changes  $1^\circ\text{C}$ , the mutual inductance

ratio error of the coil caused by the thermal expansion of coil former can be expressed as the following

$$\xi_f = \frac{M_f - M}{M} = \frac{\Delta M}{M} \approx 0.00007 + \varepsilon_f, \tag{6}$$

where  $\varepsilon_f$  is the thermal expansion coefficient of the former. When the coil former undergo a temperature changes from  $T_0^\circ\text{C}$  to  $T^\circ\text{C}$ , the mutual inductance or output voltage ratio error of the coil caused by the thermal expansion of coil former is given by

$$\xi_f = \frac{\Delta u}{u} = \frac{\Delta M}{M} \approx 0.00007 + \varepsilon_f(T - T_0). \tag{7}$$

It shows the ratio error of mutual inductance caused by the thermal expansion of coil former increases with the increase of temperature.

### 3 The thermal compensation of Rogowski coil

A. The thermal expansion effect on compensation ring.

The compensation ring, which is a thin belt loop, is embedding in Rogowski coils former. Provided  $\Delta M$  is caused by compensation ring. the ratio error of mutual inductance can be expressed as

$$\xi_c = \frac{M_t - M}{M} = \frac{\ln(\frac{b + c\varepsilon_c}{a + c\varepsilon_c})}{\ln \frac{b}{a}} - 1 = \frac{\ln[\frac{2(1+\theta) + (1+1+\theta)\varepsilon_c}{2 + (1+1+\theta)\varepsilon_c}]}{\ln(1+\theta)} - 1 \tag{8}$$

where  $\theta$  is the construction factor of the coil,  $c$  is the radii of the compensation ring,  $\varepsilon_c$  is the thermal expansion coefficient of compensation ring.

B. Discussion

The structure of former with a compensation ring is shown in Figure 5. As shown in Figure 6 per unit change in temperature, the mutual inductance error caused by the compensation ring of different materials.

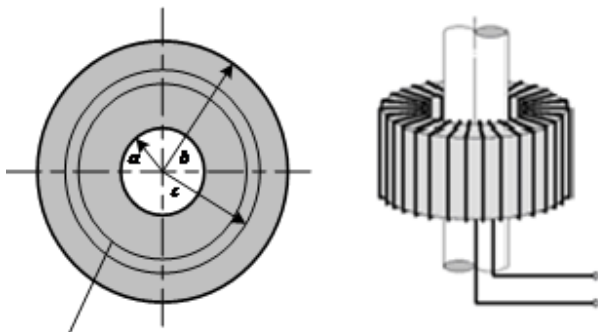


FIGURE 5 The structure of compensation ring and Rogowski coil

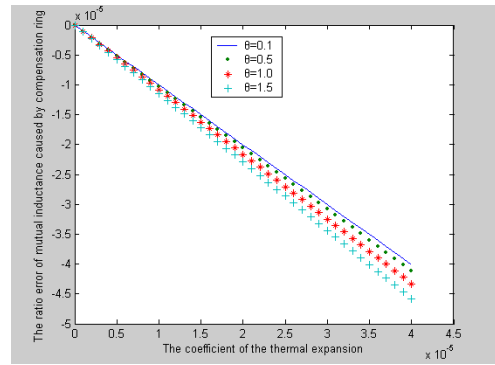


FIGURE 6 The error caused by different material of the compensation ring per unit change in temperature

From Figure 4 and Figure 6 on the one hand,  $\varepsilon_f$  is depended on  $\theta$  and  $\varepsilon$ . Namely both  $\theta$  and  $\varepsilon$  can influence the ratio error of mutual inductance. On the other hand, it is important that with the rise of temperature, the ratio error of mutual inductance caused by the coil former is positive, however the ratio error of mutual inductance caused by the compensation ring is negative. That is the compensation principle.

Under consideration of selecting  $\varepsilon_c$ ,  $\varepsilon_f$  and,  $\theta$  they can meet an equation  $\xi_f + \xi = 0$ , that is  $0.00007 + \xi_f + \xi = 0$ . In theory, the ratio error of mutual inductance caused by the thermal effect can be eliminated. If known  $\varepsilon_c$  and  $\theta$ , thus

$$0.00007 + \varepsilon_f + \frac{\ln[\frac{2(1+\theta) + (1+1+\theta)\varepsilon_c}{2 + (1+1+\theta)\varepsilon_c}]}{\ln(1+\theta)} - 1 = 0. \tag{9}$$

$\varepsilon_f$  is expressed as

$$\varepsilon_f \approx \frac{\ln[\frac{2(1+\theta) + (2+\theta)(1+\theta)\varepsilon_c}{2(1+\theta) + (2+\theta)\varepsilon_c}]}{\ln(1+\theta)}. \tag{10}$$

Equation (10) show that  $\varepsilon_f$  is calculated by the thermal expansion coefficient and the construction factor of Rogowski coil. As shown in Figure 7 the relation between  $\varepsilon_f$  and  $\varepsilon_c$ , where  $\eta$  is viewed as a set of parameters.

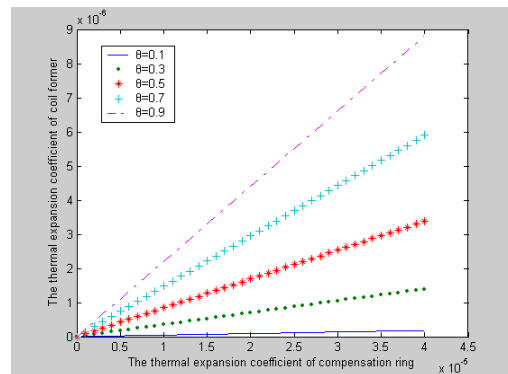


FIGURE 7 The relation between  $\varepsilon_f$  and  $\varepsilon_c$ , where  $\eta$  is viewed as a set of parameters

**4 Experimental results**

We fabricated the coil shown in Figure 8, whose parameters of configuration are listed in Table 1. The mutual inductance is calculated by (2). The current through the primary conductor is 10A 3000Hz, which is obtain by the method called equal ampere-turns. The experiment measurement system is mainly based on the following instrument and setting:

- 1) A sinusoidal AC current source WYP-4, which is adjustable in frequency from 20Hz to 20000Hz and current amplitude from 0A to 5A.
- 2) The constant output current generated by WYP-4 is monitored by a current meter.
- 3) The induced voltage from the coil is measured by KEITHLEY 2000 Multi meter with a SH3-DT9508B.
- 4) The induced voltage wave is monitor by an oscilloscope Tektronix TDS1012 in order to keep it from the case of distorted current wave.

We test Rogowski coil with rectangular cross-section Rogowski coil, the experiment data are shown in Table 2.

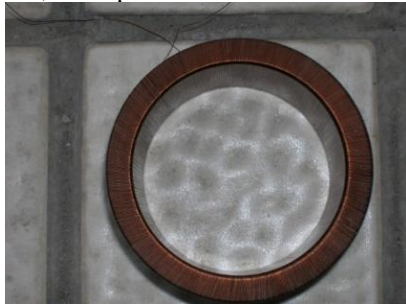


FIGURE 8 Rogowski coil with rectangular cross section

TABLE 2 The experimental data of Rogowski coil with rectangular cross section

$T /(^{\circ}C)$	$e/(mv)$	$\frac{\Delta u}{u}$ (calculated)	$\frac{\Delta u}{u}$ (measured)
30	175.24	0.001516	0.002976
20	174.72	0	0
-20	173.27	-0.006065	-0.008299
-30	172.85	-0.007581	-0.010703

It is obvious that the ratio error calculated of the output voltage is lower than that from measurement in Table 2. That is to say the error caused thermal expansion effect on the coil is a limitation of measurement precision. Because of the limitation of the interference of external magnetic field and the equipment in laboratory, the measured deviation from the calculated.

We also test Rogowski coil with and without compensation ring respectively. The experiment data are shown in Tables 3 and 4.

TABLE 3 The experimental data of Rogowski coil without compensation ring

Temperature °C	Output Voltage mV	$\xi$	
		Calculated	Measured
30	175.75	0.000 8	0.0038
20	175.08	0	0
-20	173.32	-0.003 2	-0.0101
-30	172.93	-0.004 0	-0.0123

TABLE 4 The experimental data of Rogowski coil with compensation ring

Temperature °C	Output Voltage mV	$\xi$	
		Calculated	Measured
30	175.24	-0.000 166 4	-0.000 85
20	174.72	0	0
-20	173.27	0.000 665 5	0.001801 999 1
-30	172.85	0.000 831 9	0.001597

It is obvious that with the increase of temperature, the ratio error of mutual inductance caused by the coil former is positive, however the ratio error of mutual inductance caused by the compensation ring is negative. this is consistent with the theoretical analysis. The experimental results show that the ratio voltage error tendency with temperature is the same as the error tendency from theoretical analysis and the limitation of measurement precision relates with coils thermal expansion; It may eliminate the thermal expansion effect on Rogowski coils to embed a compensation ring in Rogowski coils former.

The deviation between the measured and the calculated is caused by many factors: 1) because this article produced by the coil winding by hand, in the process of production due to the limitation of technological level, make the coil size exists a certain deviation.2) The accuracy of the measuring instrument is not enough and the reading error of the measurement. 3) The influence of interference and electromagnetic interference may also affect the accuracy of measurement.

**5 Conclusions**

The measured precision of Rogowski coil is mainly depended on the thermal coefficient of winding wire and material. In order to improve the measured precision of Rogowski coil we must eliminate or reduce the thermal expansion effects on the coil. It may eliminate the thermal expansion effect on Rogowski coils to embed a compensation ring in Rogowski coils former.

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Authors	
	<p><b>Honglong Zhang, 23 Oct 1979, Baoding, Hebei province, China</b></p> <p><b>Current position, grades:</b> Master graduate student.  <b>University studies:</b> Yansha University.  <b>Scientific interest:</b> current transformer and sensing technology.</p>
	<p><b>Bin Hong, 23 Jan 1979, Liaocheng, ShanDong province, China</b></p> <p><b>Current position, grades:</b> Master graduate student.  <b>University studies:</b> ShanDong University of Science and Technology.  <b>Scientific interest:</b> computer science and technology.</p>
	<p><b>Yuan Gui, 26 Apr 1971, Zhangjiakou, Hebei province, China</b></p> <p><b>Current position, grades:</b> Master graduate student.  <b>University studies:</b> TianJin University.  <b>Scientific interest:</b> Electrical and intelligent building.</p>
	<p><b>Yang Xiaoqing, 07 May 1965, Zhangjiakou, Hebei province, China</b></p> <p><b>Current position, grades:</b> Master graduate student.  <b>University studies:</b> Beijing University of Technology  <b>Scientific interest:</b> electric engineering.</p>