Application of fractal theory to dam deformation forecast

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Abstract

Based on the safety observation data of dam, the establishment of the prediction model of dam deformation is very important for safe operation of the reservoir. The early deformation curve of the dam has self-similar fractal feature. The fractal interpolation function can be applied to not only processing the dam prototype observation data but also forecasting the rule of early dam deformation. In this paper, the reservoir dam deformation can be analysed and predicted by the fractal interpolation function. Analysis shows that, the method for dam deformation prediction of maximum error is 8.0%. Therefore, regarding the half-year short-term forecast, the forecast result obtained from fractal interpolation function method could be reliable.

Keywords: dam, the fractal interpolation function, forecast

1 Introduction

As a water retaining structure, dam can bring social and economic benefits, but its potential safety problem also brings great threat to people's lives and properties. Although china's government has paid increasing attention on the dam safety, the security situation of the dam is still not optimistic [1-2]. Since China's vast areas are densely populated, dam-breaking in the remote region could still cause tremendous casualties and property losses. Therefore, establishing dam deformation and stress change forecast models has great significance in forecasting the dam security for some time to come, timely warning and taking effective remedial measures before the accident happens so as to minimize the disaster loss.

At present, the forecast model established by analysing the dam observation data mainly includes statistic model, grey theory prediction model, neural network method and so on. These forecast models demand to establish corresponding multi-parameter mathematical equations based on the dam operation test data. During application, the determination of the parameters could have an important effect on the forecast result. The fractal interpolation function put forward by Barnsley in 1986, the numerical value of a time-varying random variable could be forecasted as long as a parameter (scale factor) is determined. The method is especially suitable for forecasting dam deformation. The fractal theory has outstanding advantages in revealing all kinds of universal rules in dam deformation and other complicated phenomenon, so that it provides a new method for processing the dam prototype observation data and predicting dam deformation.

2 Method and forecast model

2.1 FRACTAL THEORY

For the given closed interval $I=[a,b]$, supposed that $a=x_0<x_1<\ldots<x_N=b$ is a partition of *I*, wherein *N* is more than or equal to 2, and $y_0,y_1,\ldots,y_N$ is randomly a group of real numbers, $K=I\times R$ [3], $I_i=[x_{i-1},x_i], i=1,2,\ldots,N$, supposed that L_i is a compression homeomorphism of $I\to I_i$, L_i as

$$L_i(x_0)=x_{i-1},\ L_i(x_N)=x_i\,. \tag{1}$$

If $0<l_i<1$, and then

$$|L_i(u_1)-L_i(u_2)|\le l_i,|u_1-u_2|, \forall u_1,u_2\in I\,. \tag{2}$$

Supposed that F_i is the continuous function of $K\to R$, F_i as

$$F_i(x_0,y_0)=y_{i-1},\ F_i(x_N,y_N)=y_i\,. \tag{3}$$

If $0\le q_i<1$, and then

$$|F_i(u,v_1)-F_i(u,v_2)|\le q_i|v_1-v_2|,\ \forall u\in I, v_1,v_2\in R\,. \tag{4}$$

Define the mapping $\omega_i: K\to K$:

$$\omega_i\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}L_i(x)\\F_i(x,y)\end{pmatrix}, i=1,2,\ldots,N \tag{5}$$

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and then $\{K;\omega_i, i=1,2,\ldots,N\}$ constitute an iterated function system.

Regarding the continuous function f in I (given closed interval), G is the image of f, $G=$ Graph $(f)=\{(x,f(x)), x\in I\}$ is the Invariant set of the iterated function system $\{K;\omega_i, i=1,2,\ldots,N\}$, that is to say, $G=\bigcup_{i=1}^{N}\omega_i(G)$ and, $f(x_i)=y_i, i=1,2,\ldots,N$, it is considered that such f is the fractal difference function corresponding to $\{K;\omega_i, i=1,2,\ldots,N\}$ (FIF) [3].

Supposed that the data set $(x_i,y_i): i=1,2,\ldots,N$ is given, the process for constructing the iterated function system is shown as below. When $L_i(x)$ and $F_i(x,y)$ are linear functions, formula (5) could be expressed as below:

$$\omega_i\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}a_i & 0\\c_i & d_i\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}e_i\\f_i\end{pmatrix}, i=1,2,\ldots,N \quad (6)$$

Then $L_i(x)=a_ix+e_i$, $F_i(x,y)=c_ix+d_iy+f_i$. According to formulae (2) and (4), $0<|a_i|<1$, $|d_i|<1$.

The following equation set could be derived from formulae (1) and (3).

$$\begin{cases}a_ix_0+e_i=x_{i-1}\\a_ix_N+e_i=x_i\\c_ix_0+d_iy_0+f_i=y_{i-1}\\c_ix_N+d_iy_N+f_i=y_i\end{cases} \quad (7)$$

2.2 FRACTAL PREDICTION METHOD

The dam deformation curve has the self-similarity. The settlement of the observing point could be a random process. According to the principle of fractal interpolation function, supposed that the settlement is a continuous self-similar random process $Y=\{Y(t), t\in T\}$, namely $Y(t)=\mu^{-H}y(\mu t)$, $t\in T$, $\forall\mu>0$, $0\le H<1$ H is Hurst coefficient, which has great significance in indicating the degree of self-similarity. If the Hurst coefficient is larger, closer to 1, the self-similarity (fractal characteristic) of the system would be more distinct.

Supposed that the accumulated settlement value of the dam observation point is $\{(t_i,y_i), i=0,1,\ldots,N\}$, $t_i\in[t_0,t_N]$, and $y_i\in[a,b]$, the following equations could be derived according to the fractal difference formula: $L_i(t)=t_{i-1}+(t_i-t_{i-1})(t-t_0)/(t_N-t_0)$, $F_i(t,y)=c_it+Hy+f_i$, $\omega_i(t,y)=(L_i(t),F_i(t,y))$, $i=1,2,\ldots,N$. An iterated function system could be constructed in this way. The attractor is the image of the desired fractal interpolation function *f(x)*.

Afterwards, the fractal interpolation function is extrapolated so as to forecast the dam deformation based on the existing dam observation data. The data could be extrapolated via the following method by using the interpolation function: the fractal interpolation function within the interval $[t_0,t_N]$ could be obtained from the fractal interpolation formula and then it is extrapolated. The iterated function within the interval $[t_N,t_{N+}]$ could be defined, namely,

$L_{N+1}(t)=t_N+(t_{N+1}-t_N)(t-t_0)/(t_{N+1}-t_0)$,

$F_{N+1}(t,y)=c_{N+1}+Hy+f_{N+1}$, wherein,

$c_{N+1}=(y_{N+1}-y_N-H(y_{N+1}-y_0))/(t_{N+1}-t_0)$,

$f_{N+a}=y_N-Hy_0-c_{N+1}t_0$ [4].

3 Application of fractal prediction method

TABLE 1 Observation data about deformation at the top of the earth dam of a reservoir

Time interval (month)	Settlement value within the time interval (mm)	Accumulated settlement value (mm)
1	0	0
2	23	-23
3	15	-38
4	30	-68
5	32	-100
6	18	-118
7	1	-119
8	-5	-114
9	3.6	-117.6
10	2.1	-119.7
11	0.3	-120
12	8.3	-128.3
13	4.4	-132.7
14	2.7	-135.4
15	-2.4	-133

The observation curve obtained by time series method is shown in Fig. 1.

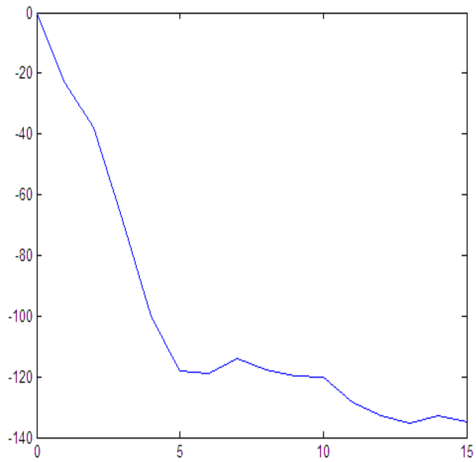

FIGURE 1 Dam top deformation curve (time series method)

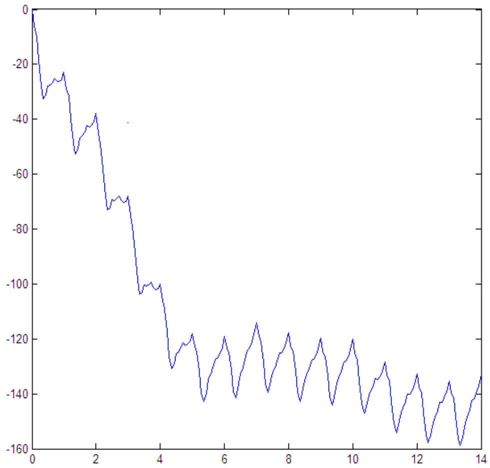

FIGURE 2 Dam top deformation curve (fractal geometry method)

Fig. 2 is the dam top deformation curve fitting by the interpolation function. Due to the limitation of the sampling frequency and the data storage capacity, the dam deformation observation data available is limited and intermittent discrete data. The fitting of original continuous data is realized via mathematic method. Obviously, the deformation curve obtained via fractal method better fits the unstable dam deformation in early period. The fractal method is mainly adopted for obtaining the dam deformation condition in the future according to the observation data.

TABLE 2 Comparison between predicted value and measured value

Time period (month)	Measured value (mm)	Predicted value	Relative error
16	-134.7	-137.7	2.20%
17	-134.1	-140.6	4.80%
18	-134.6	-143.4	6.50%
19	-135.2	-146.1	8.00%

From Table 2, it can be observed that the dam deformation is predicted by using fractal interpolation function; the error of the forecasting result will become larger along with the increase of the time period. This is related with the selected scale factor d_i , whose optimization needs further study. The maximal error of the method used in this project is 8.0%. Therefore, regarding the half-year short-term forecast, the forecast result obtained from fractal interpolation function method could be reliable.

4 Conclusion

Considering that the early deformation curve of the dam has self-similar fractal feature, the fractal interpolation function can be applied to not only processing the dam prototype observation data but also forecasting the rule of early dam deformation, with the forecast precision well consistent with the measured value. Therefore, the fractal interpolation function is particularly suitable for forecasting the dam deformation.

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