Control model of bus priority at signal intersection considering green loss equilibrium

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Abstract

To equalize the side effect that caused by the delay due to bus priority at intersections, three methods for green loss equilibrium to achieve bus priority were studied, including green extension, red truncation and phase insertion. Based on the assumption that vehicles arrives linearly, delay analysis were made at single priority request of non-saturated intersection when equilibrium is lost. Computing formula of intersection delay was presented with consideration of green light time non-priority phase loss and make-up during green light time. A timing optimization model was established which is known as normal traffic at rest phases, while objects to a function that maximize average delay at intersection. Based on Frank-Wolfe algorithm, solution came out under the diagonalized design. Instance analysis indicates the timing optimization model effectively balanced green loss at phases of non-priority, while deduced the amount of delaying at intersections. New delay calculation more explicitly described the delay change at intersections of bus priority.

Keywords: traffic engineering, bus priority signal intersection, control, timing optimization model, delay, equilibrium, green loss

1 Introduction

Signal priority is an effective way to improve public traffic efficiency, as well as increasing the attraction of bus. Per the different signals that bus requires, three preprocess signals of different characteristic were proposed by Wu [1, 2]. Delay analyses of prior and non-prior vehicles based on these three signals were made. A simulation method was used by Wahlstedt [3, 4] to analyze the how public traffic signal priority affects at signalized intersection. An analytic method was used by Liu [5] for delay analysis on vehicles at priority and non-priority approaches at the circumstance of green extension and bringing forward the green light beginning time. A delay calculation model to average vehicle delay at the self-adaptive signal control condition based on Markov character of dynamic traffic at different approaches were proposed by Jiang [6], at the basis of fixed number theory. A comparison analysis was done by Keita [7] objected to delay calculation methods of Incremental Queue Accumulation (IQA) and Highway Capacity Manual (HCM). Liu Guangping [7] analyzed saturated and unsaturated status of intersection approach. Took the max capacity of intersection into consideration, an optimized model of signal planning which targeted to minimize delayed vehicles at intersections in a cycle was established by Liu[10], while differential bacterial foraging algorithm was adopted as a resolution. Considering the affection on arterial signal progression caused by bus prior strategy, a method that makes arterial signal progression as a higher level while bus prior as a lower one was proposed by Wang Dianhai[11]. In which way, extending the green light time or switching to it earlier can prior public traffic signal. Systematic benefit of bus priority and corresponding side effects were considered in the self-adaptive bus priority control system based on rules and optimization by Wang Zhengwu [12]. Phase sequence and timing parameters were optimized as well along with the double-layered planning model. Concept of Green Light Demand Degree (GLDD) was proposed by Liu Zupeng [13] based on the demands of keeping green light and requirement of green light at green light and red light phases. Calculation method of GLDD in both phases was designed. A phase switching decision process was proposed based on GLDD, based on which, bus signal priority control came out.

Amongst the above studies on bus priority intersection timing optimization, main objects were focused on optimizing priority decision and timing of it. Signal cycles are changed along with the changes in priority decision. Considering this change, it's not good to coordinative control at intersections. Without changes of original cycles, keeping balance of the side effect caused in priority decision in a whole cycle was proposed in this article. Equilibrium parameters are optimized, while also Green Loss Equilibrium methods in green extension, red truncation and phase insertion were resolved. Delay analysis calculation was carried out. Base on it, the timing priority model which objected to minimize the amount of delaying at intersections was established, with a resolving algorithm and instances analyses.

2 Bus priority control strategy of induction

Bus priority control strategy of induction is composed of bus detection devices, wireless communication system, data control system and intelligent signal controller. As shown in Figure 1, when a bus passes upstream detection point, the detecting node is accepting vehicle information sent from dispatching system. Then the information is transferred to data control centre for an initial processing. Afterwards, decision control is made in intelligent signal controller.

Colour of the traffic light is adjusted in this way to make the bus get the priority. In Figure 1, L is the distance between detection and intersection approach, $\bar{V}$ is the speed, ΔT is the time distance between the bus and intersection approach.

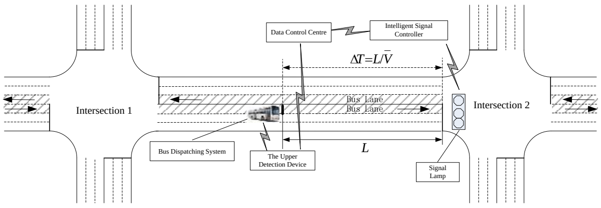

FIGURE 1 Induction control of bus priority intersection

Bus priority decision made and green loss equilibrium in the phases costs some responding time. If ΔT is too short, the priority decision is not guaranteed. And the sudden change of traffic light at intersections could result in traffic chaos because it cannot be emptied in time at intersections, which then is dangerous for pedestrians. Moving the detection point to a backward intersection can ensure enough responding time for control system, which helps coordination in a whole cycle for bus priority. After detecting the bus arrival, a whole bus priority cycle is carried out at the intersection, which can prevent from a sudden signal changing, and protect pedestrians. Counting down at intersection is not affected in the meantime.

Bus priority strategy is causing green loss to non-prior phases. They are truncated in the process of equilibrating green loss. But the truncation is not good for improving green usage efficiency, while also doesn't coincide with the continuous between actual phases. Therefore, while it's truncated in non-prior phase, that of the next phase should be extended. Assuming t stands for the time the bus arrives, is effective green light in a phase, g_{ext} is the green light extending time, g_{min} is the minimum green light, r is the effective red light, r_s is red truncation, g_{tr} is the time a phase is inserting, $g_{tr\text{-}max}$ is the inserted phase max duration.

2.1 EXTENDING GREEN LIGHT

As shown in Figure 2, when bus arrival is detected, current phase is green light, but the remaining green light is not long enough for the bus crossing the intersection, extending green light is used as a prior strategy in $g_{ext\text{-}max}$. $g_{ext\text{-}max}$ is determined by the minimum green light of non-prior phase. To decrease the affection on next phase caused by first priority phase, the green loss caused by it needs to be equilibrated in a whole cycle. The green light of next phase at the end of the second phase green light is extended, while the next of the third phase needs also to be extended in a sequence. In which way, green loss in phases is gradually fading. As:

$$\Delta t_2 = g_{ext} < g_{ext-\max};\Delta t_2 > \Delta t_3 > ... > \Delta t_n \quad (1)$$

$$g_i - \Delta t_i + \Delta t_{i+1} > g_{i-\min};\ i = 2,3...,n \quad (2)$$

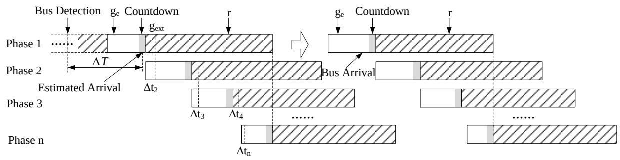

FIGURE 2 Green loss equilibrium method of green extension strategy

2.2 TRUNCATIONG RED LIGHT

As shown in Figure 3, current phase is red light when bus arrives, and the execution is to be ended, meanwhile green light of last phase is longer than minimum green light. In $r_{s\text{-}max}$, truncating red light is used as the priority strategy. $r_{s\text{-}max}$ is determined by minimum green light of non-prior phase. To equilibrate green loss of non-prior phases, previous red light time is shortened at the beginning of the n-1 phase green light, and same at the n-2 phase. In this way, green loss is equilibrated in a whole cycle, as:

$$r_s = \Delta t_{n-1} < r_{s\max};\Delta t_{n-1} > \Delta t_{n-2} > ... > \Delta t_2 \quad (3)$$

$$g_i + \Delta t_i - \Delta t_{i+1} > g_{i-\min} \quad (4)$$

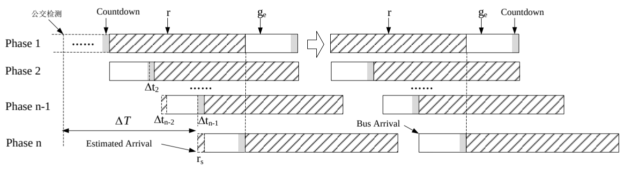

FIGURE 3 Green loss equilibrium method of red truncation strategy

2.3 INSERTING A PRIOR PHASE

As shown in Figure 4, when the bus arrival is detected, current phase is red light, and it's in the middle of execution time, minimum green light of corresponding green phase is finished while remaining green is less than inserting phase duration, prior phase insertion should be used as the prior strategy in $g_{tr\text{-}max}$. If the minimum green light of current green phase is not finished, the bus needs to wait. To avoid green light left over after inserting a phase, as well as decreasing the confliction to single non-prior phases, the end of current green light phase is used as the inserting timing. Then the green loss is afforded by two phases next to each other. Assuming phase *mid*, phase *mid*+1 are the two phases that carry phase loss, Δt_{mid} is the green loss of phase mid. To equilibrate the green loss of non-prior phases, red light of previous phase is truncated at the beginning of green light of phase mid, and green light of next phase is extended at the end of green light of phase *mid*+1. In this way, the green loss equilibrates in the whole cycle of mid, *mid*+1.

$$g_{tr} = \Delta t_{mid} + \Delta t_{mid+1} < g_{tr-\max} \quad (5)$$

$$g_k + \Delta t_{k-1} - \Delta t_k > g_{k-\min};\ \Delta t_k > \Delta t_{k-1} > ... > \Delta t_1;\ k \in [2, mid] \quad (6)$$

$$g_p + \Delta t_{p+1} - \Delta t_p > g_{p-\min};\ \Delta t_p < \Delta t_{p+1} > ...\Delta t_n;\quad p \in [mid+1, n] \tag{7}$$

In the green loss equilibrium processes as shown in Figures 2, 3, 4, inserting prior phase can be considered as an overlapping of green extension and red truncation. It is equilibrium of shortening red light gradually from *mid* to upper phases, while it's extending green light from *mid*+1 to phases below. Thus green extension and red truncation can be described as a special case of inserting a phase. When insertion happens at the end of green light, it's for extending the green light. If it happens at the beginning of green light, it's used to truncate red light. The location of inserting a phase determines three different prior strategies.

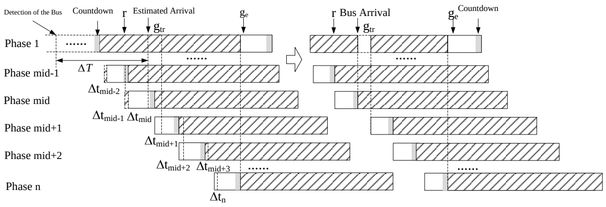

FIGURE 4 Green loss equilibrium method of phase insertion strategy

3 Analysis of vehicle delay at intersection approach

Figure 5 shows the procedures of vehicles arrival and departure at intersection approach. It is described as an overlap of A and B. Queuing and departure of static traffic stream under signal controlling is considered in process A. The stop line at approach is considered as a movable virtual line, thus departure of static traffic stream can be described as the virtual line move backward. Dynamical accumulations of the traffic stream moving to intersection stop line are considered in process B based on A. In the picture, d_i is the length of vehicle i, s_{ti} is the spacing of it.

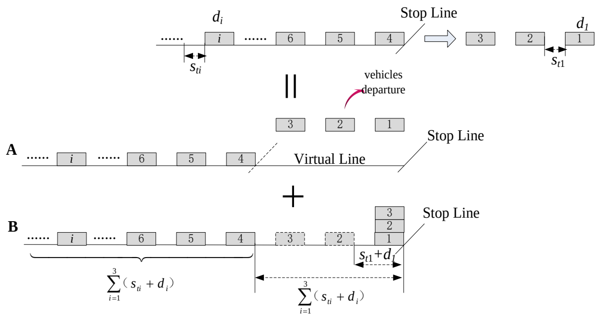

FIGURE 5 Vehicle arrival and departure procedures in the intersection imports

3.1 DELAY ANALYSIS OF PROCESS A

Figure 6 shows the process of static traffic stream leaves in a line one by one at the intersection stop line. The volume of vehicles and space headway are not taken into consideration, the stream is virtualized as a vertical sequence of particle accumulation.

It can be known in the analysis on Figure 6, delay analysis to process A is described as the geometrical analysis in Figure 7 below. In it, horizontal axis t stands for the time(s), vertical axis Q stands for accumulation of standard vehicles (pcu), q is the rate of vehicles arrival (pcu/s), s is the saturation flow rate of vehicles departure on approach (pcu/s). Square of triangle S_A is the vehicle delay in process A at intersection approach.

$$S_A = \frac{1}{2} s \frac{q \cdot r^2}{(s-q)} \tag{8}$$

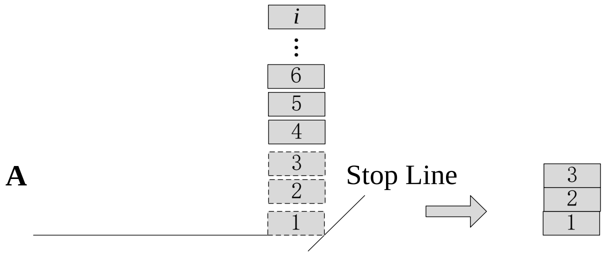

FIGURE 6 Vehicle arrival and departure procedures of process A

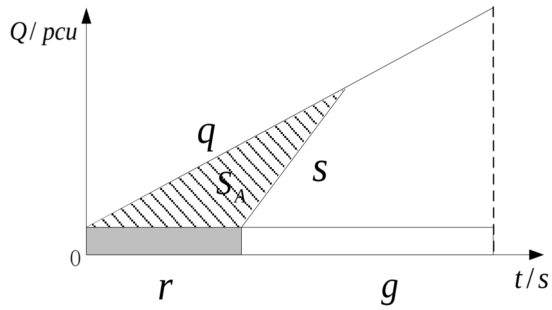

FIGURE 7 Vehicle delay analysis of process A in period

3.2 DELAY ANALYSIS OF PROCESS B

It is known from analysis of process B in Figure 5, queuing accumulations of traffic stream is causing delay as well. The longer the queue length is, the slower the starting response is. The delay can be described as a broadcasting of starting wave in the traffic queue line at approach, where v_s is the broadcasting speed of traffic starting wave, $\bar{d}$ is average vehicle length, *s* is the average space headway. Therefore, the geometrical process shown in Figure 8 can be presented as the vehicle delay analysis at intersection approach in process B.

Triangle area S_B is the vehicle delay of process B at intersection approach after analyzing Figure 8.

$$S_B = \frac{1}{2} q(\bar{d} + \bar{s})[r + \frac{q \cdot r(\bar{d} + \bar{s})}{v - q(\bar{d} + \bar{s})}]^2 \tag{9}$$

In summary, the vehicle delay D at intersection approach can be described as an overlap of process A and B.

$$D = S_A + S_B = \frac{1}{2}s\frac{q \cdot r^2}{(s-q)} + \frac{1}{2}q(\bar{d}+\bar{s}) \cdot \left[r + \frac{q \cdot r(\bar{d}+\bar{s})}{v - q(\bar{d}+\bar{s})}\right]^2 \quad (10)$$

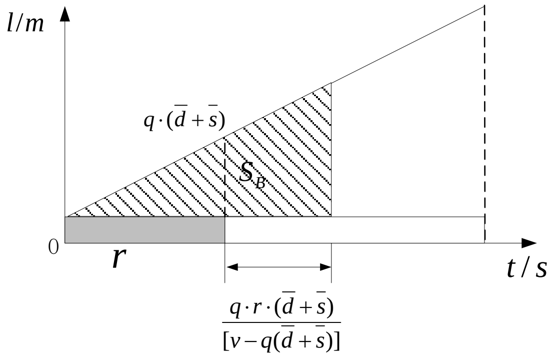

FIGURE 8 Vehicle delay analysis of process B in period

4 Vehicle delaying analysis at intersection approach under bus priority control

4.1 DELAY ANALYSIS OF PRIOR PHASE

At unsaturated intersections, assuming the traffic stream arrives linearly, bus prior is not accounting stop of the buses from social vehicles at non-prior phase. At n phase intersection, assuming there is an bus prior application in the middle of red light of the first phase, the priority strategy shown in Figure 4 is adopted, a prior phase $\Delta t_1 = g_{tr}$ is inserted into the first phase at timing $t = \sum_{i=2}^{mid} g_i - \Delta t_{mid}$. Increasing green light in the priority phase can decrease the delay. The decrement is shown in Figure 9.

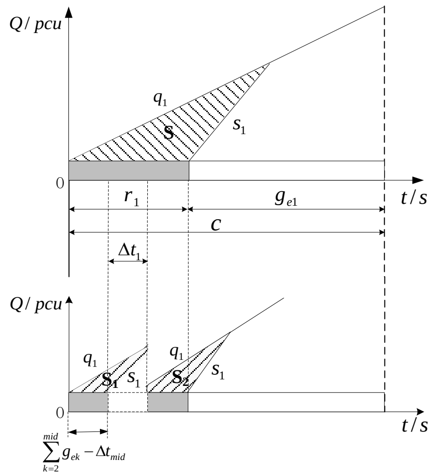

FIGURE 9 Priority phase vehicle delay analysis in period under phase insertion

In Figure 9 horizontal axis t is time(s), vertical axis Q is number of accumulated standard vehicles (pcu), q is vehicle arrival rate (pcu/s), s is the saturation flow rate of vehicles departure on approach (pcu/s). Triangle areas S is the phase delay before prior phase inserted, square $S_1 + S_2$ is the delay after phase insertion. Square $S - S_1 - S_2$ is the decreased phase delay D_{pA} in process A after prior phase is inserted.

$$t = \sum_{k=2}^{mid} g_{ek} - \Delta t_{mid} \quad (11)$$

$$S = \frac{1}{2}s_1\frac{q_1 r_1^2}{(s_1 - q_1)} \quad (12)$$

$$S_1 = \frac{1}{2}q_1(t+\Delta t_1)^2 - \frac{1}{2}s_1\Delta t_1^2 \quad (13)$$

$$S_2 = \frac{1}{2}(r_1 - t - \Delta t_1 + \frac{q_1 r_1 - s_1\Delta t_1}{s_1 - q_1})(\frac{q_1 r_1 - s_1 \Delta t_1}{s_1 - q_1}s_1 + q_1(t+\Delta t_1) - s\Delta t_1) - \frac{1}{2}s_1(\frac{q_1 r_1 - s_1\Delta t_1}{s_1 - q_1})^2 \quad (14)$$

$$D_{pA} = S - S_1 - S_2 \quad (15)$$

In a similar way, the decreased delay D_{pB} in process B after prior phase is inserted is:

$$D_{pB} = \frac{1}{2}q_1(\bar{d}+\bar{s})\left\{\left[r_1 + \frac{q_1 \cdot r_1(\bar{d}+\bar{s})}{v - q_1(\bar{d}+\bar{s})}\right]^2 - \left[t + \frac{q_1 \cdot t(\bar{d}+\bar{s})}{v - q_1(\bar{d}+\bar{s})}\right]^2 - \left[(r_1 - t - \Delta t_1) + \frac{q_1 \cdot (r - t - \Delta t_1)(\bar{d}+\bar{s})}{v - q_1(\bar{d}+\bar{s})}\right]^2\right\} \quad (16)$$

Decreased delay in a cycle after prior phase is inserted is D_1:

$$D = D_{pA} + D_{pB} \quad (17)$$

4.2 ANALYSIS TO NON-PRIOR PHASE DELAY

To non-prior phase, prior phase insertion is causing its green light truncation, so that the delay is increasing. To the kth phase, io $k \in [2, mid]$, due to an inserted phase, the red light is on at an earlier time. To make up green loss, green light of previous phase needs to be shortened. So, calculation of delay differs in phase k needs the extending delay of green loss and truncated delay of red light. As shown in Figure 10, $S_{KL} - S_K$ is the increased delay of phase k green loss in process A:

$$\Delta d_{kLA} = S_{KL} - S_K = \frac{1}{2}s_k\frac{q_k \Delta t_k(2r_k + \Delta t_k)}{(s_k - q_k)} \quad (18)$$

In a similar way, the increased delay of phase k green loss in process B is:

$$\Delta d_{kLB} = \frac{1}{2} q_k(\bar{d}+\bar{s})\left\{\left[r + \frac{q_k \cdot (r_k + \Delta t_k)(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2 - \left[r + \frac{q_k \cdot r_k(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2\right\} \quad (19)$$

Come back to the decreasing delay of phase k. To make up the green loss, green light is extended to next k-1. This period equals to the truncation of red light, so that the delay can be decreased. In the delay analysis shown in Figure 11, delay in phase k for making up green loss in process A is $S_K - S_{KO}$:

$$\Delta d_{kOA} = S_K - S_{KO} = \frac{1}{2} s_k \frac{q_k \Delta t_{k-1}(2r_k - \Delta t_{k-1})}{(s_k - q_k)} \quad (20)$$

In a similar way, decreased delay in phase k for making up green light in process B is:

$$\Delta d_{kOB} = \frac{1}{2} q_k(\bar{d}+\bar{s})\left\{\left[r + \frac{q_k \cdot r_k(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2 - \left[r + \frac{q_k \cdot (r_k - \Delta t_{k-1})(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2\right\} \quad (21)$$

Overall, increased delay in phase k, $k \in [2, mid]$, is:

$$\Delta D_k = (\Delta d_{kLA} + \Delta d_{kLB}) - (\Delta d_{kOA} + \Delta d_{kOB}) \quad (22)$$

For phase p, $p \in [mid+1, n]$, due to phase insertion, red light is extended. To make up the green loss, red of next phase needs to be truncated. So, calculation of delay differs in phase p needs the extending delay of green loss and truncated delay of red light. As shown in Figure 12, increased delay of phase p green loss in process A is:

$$\Delta d_{pLA} = S_{PL} - S_P = \frac{1}{2} s_p \frac{q_p \Delta t_p(2r_p + \Delta t_p)}{(s_p - q_p)} \quad (23)$$

In a similar way, increased delay of phase p green loss in process B is:

$$\Delta d_{pLB} = \frac{1}{2} q_k(\bar{d}+\bar{s})\left\{\left[r + \frac{q_k \cdot (r_k + \Delta t_p)(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2 - \left[r + \frac{q_k \cdot r_k(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2\right\} \quad (24)$$

In considering delay decrement in phase p, green light is extended in phase p+1 to make up with green loss in phase p, in which the duration equals to the red truncation, so that the delay is decreased. As shown in Figure 13, decreased delay in process A for making up green loss is $S_P - S_{PO}$:

$$\Delta d_{pOA} = S_P - S_{PO} = \frac{1}{2} s_p \frac{q_p \Delta t_{p+1}(2r_p - \Delta t_{p+1})}{(s_p - q_p)} \quad (25)$$

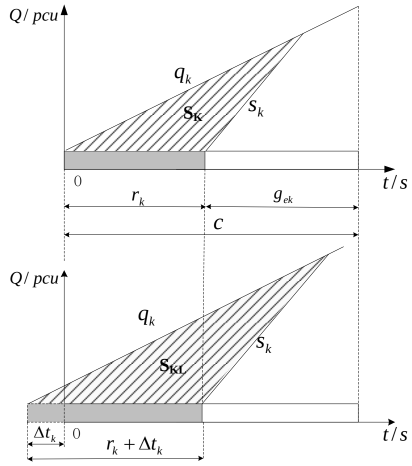

FIGURE 10 Vehicle delay analysis for green loss of phase k in period under phase insertion

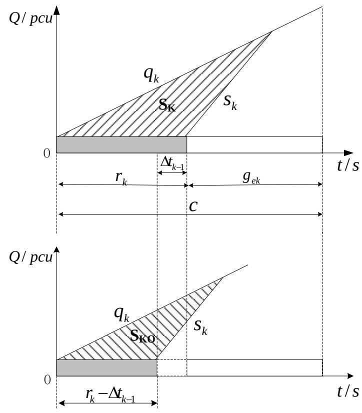

FIGURE 11 Vehicle delay analysis for loss offset of phase k in period under phase insertion

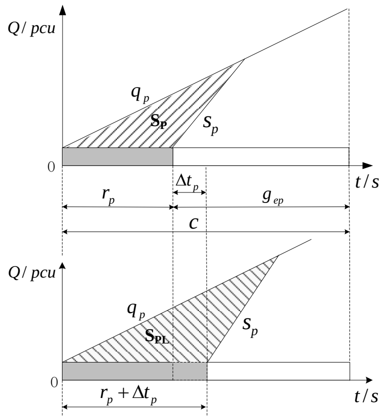

FIGURE 12 Vehicle delay analysis for green loss of phase p in period under phase insertion

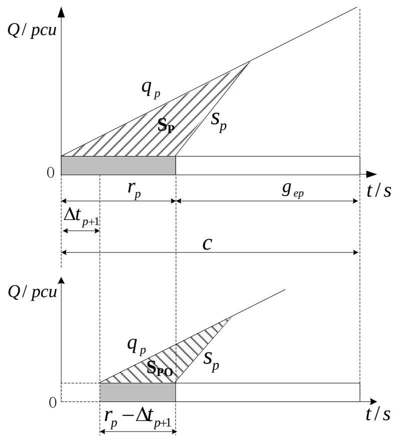

FIGURE 13 Vehicle delay analysis for loss offset of phase p in period under phase insertion

In a similar way, decreased delay in process B for making up green loss of phase p is:

$$\Delta d_{pOB} = \frac{1}{2} q_k(\bar{d}+\bar{s})\left\{\left[r + \frac{q_k \cdot r_k(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2 - \left[r + \frac{q_k \cdot (r_k - \Delta t_{p+1})(\bar{d}+\bar{s})}{v - q_k(\bar{d}+\bar{s})}\right]^2\right\} \qquad (26)$$

Overall, increment of delay ΔD_p in phase p, $p \in [mid+1, n]$ is:

$$\Delta D_p = (\Delta d_{pLA} + \Delta d_{pLB}) - (\Delta d_{pOA} + \Delta d_{pOB}) \qquad (27)$$

The incremental delay of phase i after phase insertion ΔD_i is:

$$\Delta D_i = \begin{cases} \Delta D_1 & ; i = 1 \\ \Delta D_k \big|_{k=i} & ; i \in [2, mid] \\ \Delta D_p \big|_{p=i} & ; i \in [mid+1, n] \end{cases} \qquad (28)$$

5 Timing planning optimization model

It's not equal to the phases that buses are taking larger proportion if minimizing the amount of vehicles delay in a cycle is set as the object for timing planning parameter at the intersection. Loading of buses is obviously much more than general vehicles. And decreasing the delay at intersection is rooted at decreasing delay to people, and improving travel efficiency [14]. Therefore the delay analysis at the intersections should be subjected to that of people, objected to minimizing delay for people.

To prior phases, extending green light is not only benefiting buses, some general vehicles are included as well. And to those non-prior phases affected by green loss, delaying for included general vehicles and buses is also extended. Assuming car_{ij} is the general vehicles arrival rate (pcu/s) at approach j in phase i, bus_{ij} is the corresponding bus arrival rate (pcu/s). p_{bus} is the average load of a bus in a pcu (average load of a bus divides conversion factors for equivalent cars). p_{car} is the average load of a general vehicle in a pcu. Δd_{ij} is the result calculated in formula at (28) of vehicle delay at the entrance j of phase i. The amount of people delay at an intersection in an hour is:

$$\Delta D_i^p = \sum_{j=1}^{m_i}[\Delta d_{ij}(\frac{p_{bus} \cdot bus_{ij} + p_{car} \cdot car_{ij}}{bus_{1j} + car_{1j}}) \cdot \frac{3600}{c}] \qquad (20)$$

With bus priority strategy, following timing optimization model of green loss is constructed, which targets to minimizing delay amount for people at intersections, takes guarantee of minimum green light for non-prior phases as constraint condition:

$$\max \Delta D_1^p - \sum_{i=2}^{n} \Delta D_i^p$$
$$s.t. \begin{cases} \Delta t_{mid} + \Delta t_{mid+1} = \Delta t_1 = g_{tr} \\ g_k + \Delta t_{k-1} - \Delta t_k > g_{k\text{-}\min}; k \in [1, mid] \\ g_p + \Delta t_{p+1} - \Delta t_p > g_{p\text{-}\min}; p \in [mid+1, n] \\ \Delta t_k > \Delta t_{k-1}; k \in [1, mid] \\ \Delta t_{p+1} > \Delta t_p; p \in [mid+1, n] \\ \Delta t_i > 0; i \in [2, n] \end{cases} \qquad (21)$$

In above Equation, phase insertion duration g_{tr} should make sure buses can pass through intersections, i.e. $g_{tr} > \frac{l}{v}$. In which, l is the routine distance of a bus passing the intersection, v is the average speed that a bus passes. By following Equation (21), effective green light of phases after phase insertion $\overline{g_i}$ is: $\overline{g_1} = g_1 + \Delta t_1$, $\overline{g_k} = g_k - \Delta t_k + \Delta t_{k-1}; k \in [1, mid]$, $\overline{g_p} = g_p - \Delta t_p + \Delta t_{p+1}; p \in [mid+1, n]$

6 Algorithm designing

Equation (21) is a non-linear objective function with linear constraints. Basic method of Frank-Wolfe algorithm can be used for resolution. Diagonalization technique can be used as optimized solution to unknown parameters $\Delta t_2, \Delta t_3 ... \Delta t_n$ in objective function. The target function can be changed to $\min \sum_{i=2}^{n} \Delta D_i^p - \Delta D_1^p$. Detailed steps are as following:

Step1: Initialization.

$$\Delta t_i^0 = 0; i \in [2, mid-1] \cup [mid+2, n];$$

$$\Delta t_{mid}^0 = \frac{g_{tr}}{2} \Delta t_{mid+1}^0 = g_{tr} - \Delta t_{mid}.$$

Step2: Diagonalization.

Reform the objective function as $f(\Delta t_2, \Delta t_3, ..., \Delta t_i, ..., \Delta t_n)$, initial $i = 2$, $count = 0$.

Step3: Convergence criteria setting.

$\Delta\mathbf{T} = (\Delta t_2, \Delta t_3 .. \Delta t_n)$, $\left\|\Delta\mathbf{T}^{count} - \Delta\mathbf{T}^{count-1}\right\| < gap$. If $\Delta\mathbf{T}^{count}$ is satisfied by the above convergence criterion, stop the iteration and return to $\Delta\mathbf{T}^{count}$.

Step4: Internal recycling judgment.

If $i > n$, order $i = 2$, $count = count + 1$, return to step3.

Step5: First-order Taylor approximates.

Unfold f aim at Δt_i by one-dimensional element method after fixing $\Delta t_2,\ \Delta t_3 .. \Delta t_{i-1},\ \Delta t_{i+1} .. \Delta t_n$ temporarily:

$$f(\Delta t_2,\ \Delta t_3 .. \Delta t_i .. \Delta t_n) = f(\Delta\mathbf{T}^k) + \nabla f(\Delta t_i{}^k)^T(\Delta t_i - \Delta t_i{}^k)$$

Step6: Linear programming problem solving.

Transform the linear programming problem as following:

$$\begin{aligned} &\min\ f(\Delta\mathbf{T}^k) + \nabla f(\Delta t_i{}^k)^T(\Delta t_i - \Delta t_i{}^k) \\ &s.t \quad \Delta t_i \in S \end{aligned}$$

, through which $\Delta t_i{}^{ky}$ is gotten, $\mathbf{Y}^k = (\Delta t_2{}^k,\ \Delta t_3{}^k .. \Delta t_i{}^{ky} .. \Delta t_n{}^k)$.

Step7: Convergence judgment.

If $\nabla f(\Delta t_i{}^k)^T(\Delta t_i{}^{ky} - \Delta t_i{}^k) = 0$ or $\left|\Delta t_i{}^{k+1} - \Delta t_i{}^k\right| < gap$, $i = i + 1$, else return to step4.

Step8: Line search.

Solve the linearprogramming problem as following:

$$\begin{aligned} &\min\ f(\Delta\mathbf{T}^k + \lambda(\mathbf{Y}^k - \Delta\mathbf{T}^k)) \\ &s.t \quad 0 \le \lambda \le 1 \end{aligned}$$

to get the optimal step-length λ_k . Then compute $\Delta\mathbf{T}^{k+1}$ as following: $\Delta\mathbf{T}^{k+1} = \Delta\mathbf{T}^k + \lambda_k(\mathbf{Y}^k - \Delta\mathbf{T}^k)$, return to step6.

7 Example analysis

Using an intersection of four phases as an example, Table 1 shows the phase construction and vehicle arrivals and departures. Assuming the average load on a bus in a pcu is 30, that of a general vehicle is 3, starting wave speed is 4 m/s . As shown in Figure 4, assuming a bus priority application is detected in the middle of the red light of first phase, a prior phase is insertinto it. According to the intersection scale, the timing of inserted phase is 10s.

TABLE 1 Flow data and control parameters of the intersection

Phase	Phase 1		Phase 2		Phase 3		Phase 4	
Flow direction	S-N	N-S	S-W	N-E	E-W	W-E	E-S	W-N
Effective green light in a phase /s	50		40		30		30	
Rate of general vehicles arrival (pcu/s)	0.45	0.45	0.19	0.19	0.13	0.13	0.12	0.12
Rate of buses arrival (pcu/s)	0.06	0.06	0.02	0.02	0.01	0.01	0.01	0.01
Saturation flow rate of vehicles departure (pcu/s)	1.60	1.60	0.92	0.92	0.76	0.76	0.73	0.73
Minimum green light /s	40		25		20		20	

Using above algorithm to solve Equation (21) with the data. $\Delta t_2 + \Delta t_3 = \Delta t_1 = g_{tr}$ in solving $\Delta t_2, \Delta t_3, \Delta t_4$. Replacing $\Delta t_3 = \Delta t_1 - \Delta t_2$, $f(\Delta t_2, \Delta t_3, \Delta t_4)$ is simplified as $f(\Delta t_2, \Delta t_4)$, so that the calculation could be simplified. The results are $\Delta t_2 = 2.8s$, $\Delta t_3 = 7.2s$, $\Delta t_4 = 3.1s$. Amount people delay at the intersection in an hour are 402862s.

In Table 2, Δd_i is the increment of vehicle delay in every phase in the cycle, while ΔD_i^p is the amount of increased people delay in every phase in one hour. It's known from the table that the green losses of phase 2, 3, 4 are 2.8s, 4.1s, 3.1s. Green losses are equally assigned to every phase according to its standard ability. The delay increments of non-prior phases in the cycle are 84.8s, 89.7s, 59.6s. Bus priority is carried out with a minor delay increment at non-prior phases. Green resources are transferred to green priority phase by phase continuous. Side effects caused in non-prior phases by bus priority is decreased gradually, affections of the priority strategy at the intersections are lowered down. With bus priority strategy, people delay at intersections is effectively decreased, benefits are improved.

TABLE 2 Result analysis of the optimal model

Phase	$\Delta t_i/s$	g_i/s	r_i/s	$\Delta d_i/s$	$\Delta D_i^p/s$
Phase 1	10	60	90	-1556.0	-461306.1
Phase 2	2.8	37.2	112.8	84.8	22698.6
Phase 3	7.2	25.9	124.1	89.7	21222.0
Phase 4	3.1	26.9	123.1	59.6	14523.6

8 Conclusions

Green loss equilibrium of three bus priority control strategies, which are green extension, red truncation and prior phase insertion were analyzed in this article. A delay computing was carried out at single priority request of non-saturated intersection when equilibrium is lost based on vehicle's linear arrival. An optimization model was established with minimizing the amount people delay at intersections as the objective function, which takes the normal behaviors at non-prior phases as constraints. The solution was resolved with the combination of Frank-Wolfe algorithm and diagonalization design. Buses carries priority application signals as well as general vehicles in same direction are benefit from prior phase, while the non-prior phases are affected. In green loss equilibrium, continuous relationship between phases was taken into consideration, thus the side affections of bus priority were decreased gradually, and the stabilities of phases in a cycle were maintained. The example shows that the optimization model worked for green loss equilibrium. Less losses in non-prior phases happened for bus priority. The model can work better at intersections with more complicated multi-phases. Researches of usages and developments of this model in the cases of signal priority of grouped intersections and multiple bus priority applications are worthwhile.

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