# Default assumption reasoning based on fuzzy description logics

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**Abstract**

Fuzzy description logics (DLs for short) provide a convenient tool for dealing with inconsistency and uncertainty. People can infer with uncertain and incomplete information. According to the characteristics and requirement of the knowledge representation, fuzzy DLs can play an important role in the commonsense reasoning. Default rules express concise pieces of knowledge having implicit exceptions, which is appropriate for reasoning under incomplete information. Default assumption reasoning based on fuzzy DLs is proposed. Possibility theory is used for representing both uncertainty and defeasibility. Inference service is considered in the logic and algorithms are provided for it.

*Keywords:* fuzzy description logics, default assumption reasoning, fuzzy reasoning

## 1 Introduction

Description Logic provides a logical reconstruction of the frame-based knowledge representation languages [1]. It is a significant and expressive representation and based on sound and complete constraint propagation calculi for reasoning in it. It provides a logic foundation [2] of knowledge representation and reasoning for Semantic Web [3]. DL is one of the leading formalisms for storing and manipulating knowledge in the Semantic Web. DLs allow the representation of sophisticated relations between concepts and roles; the sophistication of these relations varies, depending on the DL at hand and determines the expressive power as well as the (algorithm) complexity of reasoning in this DL.

Dealing with uncertainty has been recognized as an important problem in the recent decades. Two important classes of languages for representing uncertainty are probabilistic logic and possibilistic logic. Arguably, another important class of language for representing uncertainty is possibilistic theory [4]. Some approaches have been proposed to extend description logics with uncertainty reasoning such as reported in [5].

Typically, description logic is limited to dealing with crisp concepts. However, many useful concepts that are needed by an intelligent system do not have well defined boundaries. The need of expressing and reasoning with imprecise knowledge and the difficulties arising in classifying individuals with respect to an existing terminology is motivating research on nonclassical DL semantics, suited to these purposes. To cope with this problem, fuzzy description logics have been proposed that allow for imprecise concept description by using fuzzy sets and fuzzy relations. Umberto Straccia [6, 7] extends description logic to the fuzzy case. Fuzzy logic directly deals with the notion of vagueness and imprecision using fuzzy predicates. Therefore, it offers an appealing foundation for a generalization of description logic in order to dealing with such vague concepts.

The process of human is reasoning decision is dynamic and uncertain. We can get the conclusion under the condition of fuzzy and incomplete information. Handling exceptions in a knowledge-based system has been considered as an important issue in many domains of applications. Reiter's default logic [8] is one of the most popular formalisms for describing non-monotonic reasoning and has been extensively investigated by the community working on logical foundations of artificial intelligence. Default rules express concise pieces of knowledge having implicit exceptions, which is appropriate for reasoning under incomplete information. Handling uncertainty in a given complete information context is a need in various situations. For example, high level descriptions of dynamical systems often requires both the use of default rules expressing persistence and the processing of uncertainty due to the limitation of the available information.

Reasoning under incomplete information by means of rules having exceptions, and reasoning under uncertainty are two important type of reasoning that artificial intelligence has studied at length and formalized in different ways in order to design inference systems able to draw conclusions from available information as it is. As already said, reasoning with default rules and under uncertainty are two important research trends that have been developed quite independently from each other in AI. They indeed address two distinct problems, respectively using symbolic and numerical approaches in general.

In this paper, we extend fuzzy DLs by providing a framework for the default assumption reasoning. This paper outlines a joint handling of defaults and fuzzy DLs.

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Default assumption reasoning based on fuzzy DLs is proposed. Inference service is considered in the logic and algorithm is provided for it. This is a continuous process of adjustment to the fuzzy DLs reasoning. However, the main novelty is that these adjustments are made to non-monotonic reasoning, as defaults describe assumption which guide the reasoning process.

The rest of this paper proceeds as follows. Preliminaries on fuzzy description logics are given in Section 2. Default assumption reasoning based on Fuzzy DLs is provided in Section 3. The inference services are also given. After that, we provide algorithms for implementing reasoning problems. And the last one is the conclusion and the future work.

## 2 Preliminaries

In this section, we introduce some background knowledge about fuzzy description logics.

Description Logics are a well-known family of knowledge representation formalisms. They are based on the notions of concepts (unary predicates, classes) and roles (binary relations), and are mainly characterized by constructors that allow complex concepts and roles to be built from atomic ones. The expressive power of a DL system is determined by the constructs available for building concept descriptions, and by the way these descriptions can be used in the terminological (*TBox*) and assertional (*ABox*) components of the system.

Straccia extends description logic to fuzzy description logic with fuzzy capabilities. Due to the limitation of space, we do not provide a detailed introduction of fuzzy DLs, but rather point the reader to [6].

### 2.1 SYNTAX AND SEMANTICS OF FUZZY DLS

Similarly to crisp DL languages, fuzzy DLs concepts are defined by the following syntax rule:

$$C,D \rightarrow \mathrm{T} \mid \perp \mid A \mid \neg C \mid C \cup D \mid C \cap D$$
$$\exists R.C \mid \forall R.C$$

A fuzzy DL knowledge base consists of two finite and mutually disjoint sets. A *TBox*, which introduces the terminology, and *ABox*, which contains facts about particular objects in the application domain.

A terminology, or *TBox*, is defined by a finite set of fuzzy concept inclusion axioms of the form $A \subseteq C$ and fuzzy concept equalities of the form $A \equiv C$.

Objects in the ABox are referred to by a finite number of individual names and these names may be used in two types of assertional statements: concept assertions of the type $a:C$ and role assertions of the type $(a,b):R$, where $C$ is a concept description and $R$ is a role name, and $a,b$ are individual names.

Let $I=\{a,b,c...\}$ be a set of individual names. A fuzzy assertion is of the form $\langle a:C\theta n\rangle$ or $\langle (a,b):R\theta n\rangle$, where $\theta$ stands for $\geq,\leq,>,<$. Intuitively, a fuzzy assertion of the form $\langle a:C\geq n\rangle$ means that the membership degree of $a$ to the concept $C$ is at least equal to $n$. A finite set of fuzzy assertions defines a fuzzy *ABox* $A$. The concept of conjugated pairs of fuzzy assertions has been introduced, in order to represent pairs of assertions that form a contradiction. The possible conjugated pairs are defined in table 1, where $\phi$ represents a concept expression.

TABLE 1Conjugated pairs of fuzzy assertions

| | $\langle\phi<m\rangle$ | $\langle\phi\leq m\rangle$ |
|---|---|---|
| $\langle\phi\geq n\rangle$ | $n\geq m$ | $n>m$ |
| $\langle\phi>n\rangle$ | $n\geq m$ | $n\geq m$ |

TABLE 2 SEMANTICS OF FUZZY CONCEPTS

$$\mathrm{T}^I(a)=1$$
$$\perp^I(a)=0$$
$$(\neg C)^I(a)=1-C^I(a)$$
$$(C\cup D)^I(a)=\max\left(C^I(a),D^I(a)\right)$$
$$(C\cap D)^I(a)=\min\left(C^I(a),D^I(a)\right)$$
$$(\forall R.C)^I(a)=Inf_{b\in\Delta^I}\left\{\max\left(1-R^I(a,b),C^I(b)\right)\right\}$$
$$(\exists R.C)^I(a)=Sup_{b\in\Delta^I}\left\{\min\left(R^I(a,b),C^I(b)\right)\right\}$$

A fuzzy set $C\subseteq X$ is defined by its membership function ($\mu_C$), which given an object of the universal set $X$ it returns the membership degree of that object to the fuzzy set. By using membership functions, we can extend the notion of an interpretation function to that of a fuzzy interpretation. A fuzzy interpretation $I$ consists of a pair $\left(\Delta^I,\cdot^I\right)$, where $\Delta^I$ is the domain of interpretation, as in the classical case, and $\cdot^I$ is an interpretation function which maps concepts (roles) to a membership function $\Delta^I\rightarrow[0,1]$ ($\Delta^I\times\Delta^I\rightarrow[0,1]$), which defines the fuzzy subset $C^I$ ($R^I$). The semantics of fuzzy DL are depicted in table 2.

A fuzzy concept $C$ is satisfiable iff there exists some fuzzy interpretation $I$ for which there is some $a\in\Delta^I$ such that $C^I(a)=n$, and $n\in[0,1]$. A fuzzy interpretation $I$ satisfies a *TBox T* iff $\forall a\in\Delta^I, A^I(a)\leq D^I(a)$, for each $A\subseteq D$, and $\forall a\in\Delta^I, A^I(a)=D^I(a)$,for each $A\equiv D$. Fuzzy interpretations are also extended to interpret individual and assertions that appear in an *ABox*. For a fuzzy *ABox*, an interpretation maps, additionally, each individual $a\in I$ to some element $a^I\in\Delta^I$. An interpretation $I$ satisfies a fuzzy assertion $\langle a : C \geq n\rangle$ *iff* $C^I(a) \geq n$ , $\langle (a,b) : R \geq n\rangle$ *iff* $R^I(a^I, b^I) \geq n$ .

The satisfiability of fuzzy assertions with $\leq, >, <$ is defined analogously.

A fuzzy *ABox* $A$ is consistent iff there exists an interpretation $I$ that satisfies each fuzzy assertion in the fuzzy *ABox*. We then say that $I$ is a model of *A*.

## 2.2 A FUZZY TABLEAU FOR FUZZY DLs

In the following, the fuzzy entailment problem is reduced to the unsatisfiability problem of a set of fuzzy assertions. A tableau algorithm is used to construct a fuzzy tableau for a fuzzy $KB$ . Given a fuzzy $KB\, \Sigma = \langle T, A\rangle$ , let $S_\Sigma = \{\langle a : C \geq n\rangle\} \cup \{\langle (a,b) : R \geq n\rangle\}$.

It follows that $\Sigma \Rightarrow \langle a : C \geq n\rangle$ iff $S_\Sigma \cup \{\langle a : C < n\rangle\}$ not satisfiable.

The calculus, determining whether a set $S$ of fuzzy assertions is satisfiable or not, is based on a set of fuzzy constraint propagation rules transforming a set $S$ of fuzzy assertions into "simpler" model preserving sets $S_i$ untile either all $S_i$ contains a clash.

A set of fuzzy assertions $S$ contains a clash iff it contains either $\langle w : \perp \geq n\rangle$ with $n > 0$ or $\langle w : \perp > n\rangle$ or $\langle w : \perp < 0\rangle$ or $\langle w : \mathrm{T} \leq n\rangle$ with $n < 1$ , or $\langle w : \mathrm{T} < n\rangle$ or $\langle w : \mathrm{T} > 1\rangle$ , or $S$ contains a conjugated pair of fuzzy assertions.

Given a fuzzy assertion $\sigma$ , with $\sigma^c$ we indicate a conjugate of $\sigma$ . Concerning the rules, for each connective $\cap, \cup, \neg, \forall, \exists$, there is a rule for each relation $rel \in \{\geq, \leq, >, <\}$. The rules for the case $rel \in \{>, <\}$ are quite similar. The rules are the following:

$(\neg_\geq)$ $\langle w : \neg C \geq n\rangle \rightarrow \langle w : C \leq 1 - n\rangle$

$(\neg_\leq)$ $\langle w : \neg C \leq n\rangle \rightarrow \langle w : C \geq 1 - n\rangle$

$(\cap_\geq)$ $\langle w : C \cap D \geq n\rangle \rightarrow \langle w : C \geq n\rangle, \langle w : D \geq n\rangle$

$(\cup_\leq)$ $\langle w : C \cup D \leq n\rangle \rightarrow \langle w : C \leq n\rangle, \langle w : D \leq n\rangle$

$(\cup_\geq)$ $\langle w : C \cup D \geq n\rangle \rightarrow \langle w : C \geq n\rangle \mid \langle w : D \geq n\rangle$

$(\cap_\leq)$ $\langle w : C \cap D \leq n\rangle \rightarrow \langle w : C \leq n\rangle \mid \langle w : D \leq n\rangle$

$(\forall_\geq)$ $\langle w_1 : \forall R.C \geq n\rangle, \sigma^c \Rightarrow \langle w_2 : C \geq n\rangle$ *if* $\sigma$ *is* $\langle (w_1, w_2) : R \leq 1 - n\rangle$

$(\exists_\leq)$ $\langle w_1 : \exists R.C \leq n\rangle, \sigma^c \Rightarrow \langle w_2 : C \leq n\rangle$ *if* $\sigma$ *is* $\langle (w_1, w_2) : R \leq n\rangle$

$(\exists_\geq)$ $\langle w : \exists R.C \geq n\rangle \rightarrow \langle (w, x) : R \geq n\rangle$, $\langle x : C \geq n\rangle$ *if* $x$ *is a new* variable

$(\forall_\leq)$ $\langle w : \forall R.C \leq n\rangle \rightarrow \langle (w, x) : R \geq 1 - n\rangle$, $\langle x : C \leq n\rangle$ *if* $x$ *is a new* variable

A set of fuzzy assertions $S$ is said to be complete if no rule is applicable to it. These rules are called monotonic rules.

## 3 Default assumption reasoning

Reasoning under incomplete information by means of rules having exceptions is a very important type of reasoning that artificial intelligence has studied at length and formalized in different ways in order to design inference system able to draw conclusions from available information as it is. It is useful especially for the knowledge representation, which is popular in framework reasoning, diagnostic reasoning, and natural language processing and so on.

However, well-known results from the literature show that DLs have limitations: they do not allow for expressing default knowledge due to their inherent monotonic semantics. One needs nontrivial extensions to the semantics of description logics to express exceptional knowledge.

**Example 1**. Take, as an example, a bird ontology expressed in the fuzzy DL knowledge base:

$KB = \langle T, A\rangle$ :

$$T = \left\{ \begin{array}{l} Flier \subseteq_1 \neg NonFlier,\ Pengin \subseteq_1 Bird, \\ Pengin \subseteq_1 NonFlier \end{array} \right\}$$

$A = \{Bird(tweety)\}$

Intuitively, KB distinguishes between flying and non-flying objects. We know that penguins, which are birds do not fly. Nevertheless, we cannot simply add the axiom $Bird \subseteq_1 Flier$ to $KB$ to specify the common view that "birds normally fly", as this update will make KB inconsistent. From our bird ontology, we would like to conclude that tweety flies; and if we learn that tweety is a penguin, the opposite conclusion would be expected.

Hence, the simple ontology KB from above cannot express exceptional knowledge, an extension of the semantics of terminological knowledge was given in [9], which is an early attempt to support default logic in the domain of description logics.

Several other attempts to extend DLs with nonmonotonic features have been made based on default logics [10, 11].

In fuzzy DLs, the concept descriptions are interpreted as universal statements, which means, unlike frame languages, they do not allow for exceptions. One needs a formalism that can handle default assumptions, but does not destroy the definitional character of concept description. Default rules are useful in order to express general behaviours concisely, without referring to exceptional cases. Moreover, they only require general information to be fired, which agrees with the situations of incomplete information. With the aim to offer a user-friendly reasoned over ontologies, we consider default reasoning on top of ontologies based on fuzzy DLs, which integrate default rules and ontologies.

The process of human is reasoning decision under incomplete information can be abstracted as follows: firstly, make an assumption; secondly, reason with the assumptions and draw the conclusions in the case of default assumptions; thirdly, make an evaluation of the result. If the conclusion based on default assumptions is satisfied or consistent with current knowledge base, it is accepted and continues to make further decision reasoning. Otherwise, the assumption is denied, either given up or reassumed.

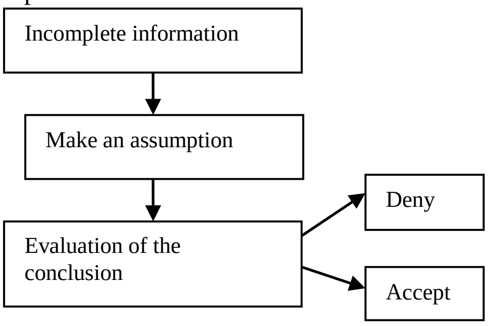


FIGURE 1 The process of default assumption reasoning

According to the process, we consider the default assumption reasoning of fuzzy DLs. First of all, the default assumption rule base is made under the incomplete information from which we can draw conclusions. Secondly, evaluation system is established. The conclusion is evaluated according to the decision goal. Finally, if the conclusion is inconsistent with the growing information, we can go back to the previous hypothesis.

## 3.1 THE FRAMEWORK OF DEFAULT ASSUMPTION REASONING

In the inference system based on fuzzy DLs, the rule base is divided into two parts: one is the general rule base and the other one is default assumption rule base. Both rule bases including knowledge base work together to draw a conclusion. The framework of reasoning is shown in figure 2.

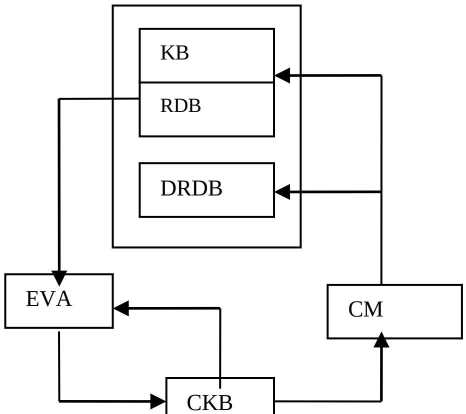


FIGURE 1 The framework of default assumption reasoning

Each part of figure 2 is defined as follows:

KB: a knowledge base of the form $\langle T, A \rangle$.

RDB: a rule base including the reasoning rules based on fuzzy DLs, which are described in section 2.2

DRDB: a default assumption rule base

CKB: a current knowledge base contains current information, which is used during the reasoning process

EVA: the evaluation mechanism, which needs a trigger

CM: the inference control mechanism, which controls the reasoning process

The basic idea and the process are described as follows:

(1) The current knowledge base CKB submits to the inference control mechanism CM the current knowledge (fuzzy assertions);

(2) Inference control mechanism CM will search for the information in the current knowledge base CKB and the rule base RDB. If there are rules matched, the inference is triggered by the reasoning mechanism. Otherwise, if there are no rules or knowledge matched in CKB and RDB, we will check the default assumption rule base DRDB. If there are some default assumption rules matched in DRDB, we will go on the hypothesis reasoning and get the conclusion. At the same time, we mark the conclusion and prepare for the tracing back.

(3) The conclusion directly reasoned by the rule base RDB will be put in the current knowledge base CKB. If it is drawn by the default assumption rule base DRDB, it needs the evaluation mechanism EVA to check it. If it is inconsistent with the current knowledge of CKB, the conclusion reasoned by default assumption is considered to be not plausible and abandon, and we will go back to the previous hypothesis state.

(4) For the fuzzy assertion of the form $\langle x : C \geq n \rangle$, if we cannot check the satisfiability of its negative form $\langle x : \neg C < 1 - n \rangle$, then the default assumption rule base DRDB is called. We assume that $\langle x : C \geq n \rangle$ be satisfiable and reason with it. Following the hypothesis we get the conclusion. After that, we call the evaluation mechanism EVA to check it.

### 3.2 UNCERTAIN DEFAULT RULES

A default rule is an expression $a \sim> b$ where $a$ and $b$ are formulas and ~> is a new symbol. $a \sim> b$ Translates, in the possibility theory framework, into the constraint $\Pi(a \wedge b) > \Pi(a \wedge \neg b)$, which expresses that having $b$ true is strictly more possible than having it false when $a$ is true.

The use of default rules has two main interests. First, it simplifies the writing: it allows us to express a rule without mentioning every exception to it. Second, it allows us to reason with incomplete descriptions of the world: if nothing is known about the exceptional character of the situation, it is assumed to be normal, and reasoning can be completed.

In order to have more expressive representation formalism, we now introduce the notion of uncertain default rule.

Definition 1 An uncertain default rule is a pair $(a \sim> b, \alpha)$ where $a$ and $b$ are concepts or roles of DLs, and $\alpha$ is the certainty level of the rule, the symbol ~> is a non classical connective encoding a non-monotonic consequence relation between $a$ and $b$.

The intuitive meaning of $(a \sim> b, \alpha)$ is "by default" if is true then has a certainty level at least equal to $\alpha$. For instance, $(bird \sim> flies, \alpha_1)$ means that a bird generally flies with certainty $\alpha_1$. It is a default rule since it admits exceptions mentioned in other rules: for instance, $(bird \wedge young \sim> \neg flies, \alpha_2)$: young birds generally do not fly. But it is also an uncertain rule since all we know is that we are in presence of a bird, the certainty level $\alpha_1$ is attached to the provisional conclusion that it flies. Thus, the $\alpha$'s provide an additional information with respect to the default rule.

The core of the treatment of uncertain default rules proposed in this paper is based on the idea of translating them into a set of uncertain rules.

Roughly speaking, default reasoning amounts to apply a set of default rules to the knowledge base describing a context.

The core of the treatment of uncertain default rules proposed in this paper is based on the idea of translating them into a set of uncertain (non defeasible) rules.

Definition 2 Let $D = \{(a_i \sim> b_i, \alpha_i), i = 1,2..n\}$ be an uncertain default rules set. Suppose $\beta_j, j = 1,2..k$ are all distinct weights appearing in $D$ such that $\beta_1 > \beta_2 > ... > \beta_k$. Let $\Sigma_D = (S_1, S_2, ..S_k)$, where $S_i = \{a_l \to b_l : (a_l \sim> b_l, \alpha_l) \in D, \alpha_l = \beta_l\}$.

Since possibilistic inference suffers from the drowning problem, we consider a drowning-free variant of possibilistic inference, called linear order inference. Uncertain default rules are stratified by the weights. Reasoning from a set of exception-tolerent default rules in presence of incomplete knowledge first amounts to select default rules. The selected set of rules should focus on the current context describing the particular incomplete information situation that is considered, and then this set of rules can be applied to this information situation in order to draw plausible conclusions. When new information is available on the current situation, these conclusions may be revised at the light of more appropriate default rules. The selection problem is solved in practices by rank-ordering the default rules in such a way that most specific rules whose conclusion may conflict with the conclusion of more general defaults, receive a higher level of priority. Clearly, the level of priority of a particular rule depends on the whole set of default rules which are considered.

### 3.3 ALGORITHMS FOR INFERENCE IN THE FRAMEWORK

We give algorithms for the inference in the framework based on fuzzy DLs.

(1) Algorithm 1 computes the fuzzy membership degree $n$ of the fuzzy assertion $\langle x : C \ge n \rangle$. $i.\alpha$ of the algorithm 1 is the fuzzy membership degree of the prequisite of the default rule.

Algorithm 1. Function FMA

Data: $KB = \langle T, A \rangle$; a DL concept $\langle a : C \ge n \rangle$

Result: The membership degree $n$ associated with a query $C(a)$

begin
Foreach $r$ in RDB and DRDB do
    if $\langle a : C \ge n \rangle$ matching $r$.rule
        if $r$.rule is default rule in DRDB
        then flag=1
else flag=0
        if flag==1 then
            if $r.\alpha < 1$ then $n = r.\alpha$
            else $n = 1$
else according to the RDB to compute $n$
end

(2) Algorithm 2 of the inference control mechanism FCM

$\mu$ of the algorithm 2 is the threshold.

Algorithm 2 Function FCM

Data: $CKB = \{\langle \phi_i, \alpha_i \rangle : \alpha_i \in [0,1], i = 1,...,k\}$, where $k$ is the number of fuzzy assertions in the current knowledge base $CKB$; a fuzzy DL concept $\langle a : C \ge n \rangle$

Result: The update of knowledge base $KB$

begin
if $\langle a : C \ge m \rangle$ exists in $CKB$ then
    add $\langle a : C \ge \min(m, n) \rangle$ to knowledge base $KB$
else FMA($\langle a : C \ge n \rangle$)
if $n > \mu$ then
if flag==1 then EVA($\langle a : C \ge n \rangle$)

else add $\langle a : C \geq n \rangle$ to knowledge base $KB$
end

(3) Algorithm 3 of the evaluation mechanism EVA
Alogrithm 3 Function EVA( $\langle a : C \geq n \rangle$ )
Data: $KB = \langle T, A \rangle$ ; a fuzzy DL concept $\langle a : C \geq n \rangle$
Result: The update of knowledge base $KB$
begin
check $\langle a : C \geq n \rangle$ in knowledge base $KB$
if it is consistent then add it to knowledge base
else give up the result
end

### 3.4 DISSUCTIONS ON THE CONSISTENCY

If the conclusion is drawn from the default assumption rule or part 4 of section 3.1, we will mark a sign in the algorithm FMA and FCM. When the conclusions tagged are inconsistent with ones from EVA, we will delete the tagged conclusions to make assure the consistency of the knowledge base.

To get a sense of how representations work within this formalism consider the following example.

**Example 2.** Suppose we have a knowledge base $KB = \langle T, A \rangle$

$$T = \begin{Bmatrix} Penguin \subseteq_{0.8} \neg fly, Ostrich \subseteq_{0.8} \neg fly, \\ Sparrow \subseteq_1 bird, Penguin \subseteq_1 bird \end{Bmatrix}$$

$$A = \varnothing$$

$$DRDB = \begin{Bmatrix} (bird \wedge Penguin \sim> \neg fly, 0.8), \\ (bird \wedge Ostrich \sim> \neg fly, 0.8), \\ (bird \sim> fly, 0.6) \end{Bmatrix}$$

Now the query is "Can Sparrow fly?", which means that we should judge the entailment problem of the concept "Sparrow" and "fly", namely $Sparrow \subseteq_n fly,\ n \in [0,1]$. At first, we change $T$ and $A$ into the form of fuzzy constraint system:

$$S = \begin{Bmatrix} x : \neg Penguin \cup fly \geq 0.8, \\ x : \neg Ostrich \cup fly \geq 0.8, \\ x : \neg Sparrow \cup bird \geq 1, \\ x : \neg Penguin \cup bird \geq 1 \end{Bmatrix};$$

By the definition 2, the uncertain default rules of $DRDB$ are stratified: $DRDB = (S_1, S_2)$, where

$$S'_1 = \begin{Bmatrix} (bird \wedge Penguin \sim> \neg fly, 0.8), \\ (bird \wedge Ostrich \sim> \neg fly, 0.8) \end{Bmatrix}$$

$$S'_2 = \{(bird \sim> fly, 0.6)\}$$

For simplicity, we translate them directly into a set of uncertain rules:

$$S_1 = \begin{Bmatrix} (bird \wedge Penguin \to \neg fly, 0.8), \\ (bird \wedge Ostrich \to \neg fly, 0.8) \end{Bmatrix}$$

$$S_2 = \{(bird \to fly, 0.6)\}$$

Secondly, the problem is changed into the fuzzy assertion satisfiability or default assumption satisfiability check. We add $\{x : Sparrow \cup \neg fly < n\}$ to the fuzzy constraint system *S*, the propagation rules of *RDB* are applied as follows:

(1) $\langle x : Sparrow \cup \neg fly < n \rangle$
(2) $\langle x : Sparrow < n \rangle$ $\quad \cup_<$
(3) $\langle x : \neg fly < n \rangle$ $\quad \cup_<$
(4) $\langle x : \neg Sparrow \cup bird \geq 1 \rangle$
(5) $\langle x : \neg Sparrow \geq 1 \rangle$ $\quad \cup_\geq$
(6) clash (2) and (5)
(7) $\langle x : bird \geq 1 \rangle$ $\quad \cup_\geq$
(8) $\langle x : fly \geq 0.6 \rangle$ the rule of $S_2$
(9) clash if $n < 0.6$ (3) and (8)

It follows that $Sparrow \subseteq_n fly$ if $n \geq 0.6$. Therefore, if the threshold of $\mu$ is set $0.5$, we can draw a conclusion that "Sparrow can fly in general".

## 4 Related work

There have been some works in fuzzy DLs, such as the work reported in [12-16]. But there has been very few works handling both defeasibility and uncertainty. Our main goal is to provide a framework for fuzzy default reasoning. The formalism used in this paper is an extension of fuzzy DLs as introduced by Straccia.

In [17] a technique is described for assigning a preference semantics for defaults in terminological logics, which uses exceptions and therefore has some similarities to our work. They draw a distinction between strict inclusions (TBox statements of the form) and defaults, which is interpreted as "soft" inclusions. We also make use of the notion of the stratification of default theories studied in detail by Choleuinski [18]. In our framework, the uncertain default rules are stratified by membership degree.

Using an uncertain framework in order to describe an evolving system has been done by many authors, for instance in a probabilistic setting. But reasoning in this setting implies to dispose of many priori probabilities; this is why using defeasibility may help to reduce the size of information for representing the system. In this sense our framework is more expressive. However, the use of default rules requires more complex approach. The computation of extensions is difficult and we do not have to take account for it.

An altogether different approach is the explicit introduction of nonmonotonicity into DLs, usually some variant of default logic. See [19] for an overview. But this extension is limited to classical description logics, fuzzy characteristic is not considered.

Nicolas, Garcia and Stephan [20] also present an approach that deals with defeasibility and uncertainty in a possibilistic framework. But, they combine possibilistic logic with Answer Set Programming rather than using the same setting for default and uncertainty handling. Dupin [21] has introduced a new rewriting algorithm in applications to handle uncertain default rules. This work is different from the work [22] on extending fuzzy DLs with default rules, where classical default rules are directly attached with the tableau algorithm. The classical default rules are more complex while the use of default assumption rule allows us to express a rule without mentioning every exception to it.

## 5 Conclusions and future work

Based on the fuzzy description logic with default reasoning, we construct a knowledge base system that incorporates *TBox*, *ABox* and default rules. We have proposed a default assumption extension for fuzzy description logics and provided corresponding algorithms in this paper. We provide a theoretical framework allowing us to study the feasibility of applying default assumption in fuzzy DLs. The model described here shows an efficient way to use uncertain default reasoning as a tool for fuzzy DLs. This topic is interesting because the problem of dealing with uncertainty is closely related to the problem of nonmonotonic reasoning and possibility theory has been widely accepted as an important formalism to deal with uncertainty.

We have shown how the structure of fuzzy DL languages can be exploited to define basic knowledge base. Our focus was on the formal semantics of knowledge representation, although we also provided high-level decision procedures. The next step is the development of tableaux-based algorithms for implementing the strategies outlined in the paper. The combination of default assumption rules and fuzzy DLs enable us to capture knowledge about vagueness and defeasibility. Furthermore, the incorporation of default assumption allows us to encode and reason under incomplete information. Stratification strategy is considered in the uncertain default rule set which is more convenient for default reasoning. We have not considered the complexity analysis of algorithms but leave them as future work.

An obvious question to consider is whether any additional structure, such as the specification of role hierarchies, can be exploited further to modify the knowledge representation in appropriate ways. Finally, the management of other notions of incoherence, such as concept unsatisfiability, is currently the topic of further investigation.

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