The optimal dynamic robust portfolio model

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Abstract

This paper is concerned with the optimal dynamic multi-stage portfolio of mean- dynamic var based on high frequency exchange data with the constraint of transaction costs transaction volume. The proposed solution approach is based on robust optimization, which allows us to obtain a worst best but exact and explicit problem formulation in terms of a convex quadratic program. In contrast to the mainstream stochastic programming approach to multi-period optimization, which has the drawback of being computationally intractable, the proposed setup leads to optimization problems that can be solved efficiently.

Keywords: dynamic portfolio, mean-var, robust, high frequency exchange

1 Introduction

Markowitz [1] is the first to formulate the model for maximizing the expected return and minimizing its risk. He only considered the case of a single-period investment. However, investment behaviour, especially the investment behaviour of institutional investors are often long. For a long-term investor, he will adjust portfolio positions with the investment environment changes timely, rather than portfolio immutable and frozen early construction to keep to the investment plan period. With the development of computer, obtaining high frequency data more convenient is benefit to establish of dynamic investment strategy. So in order to reformulate the single stage asset allocation problem, a multi-period framework is a decision process has been developed by using multi-period stochastic programming, see for example [2-4] Kall (1976), Wallace (1994) and Breton (1995). The academics modelled the mean or the variance of total wealth at the end of the investment horizon as either linear stochastic programming or quadratic stochastic programming in Gulpinar et al. (2002, 2003) [5-6]. Inoue, and Wang (2010) [7] proposed a dynamic portfolio selection optimization with bankruptcy control for absolute deviation model.

Decision problems arising in engineering, finance, logistics etc. are usually dynamic and affected by uncertainty. However, in optimal portfolio model, it requires the knowledge of both mean and covariance matrix of the asset returns, which practically are unknown and need to be estimated. The standard approach, ignoring estimation error, simply treats the estimates from the history data as the true parameters and plugs them into the optimal portfolio optimization model derived under the mean–variance framework. That is, using known information replace the unknown information. Moreover, just as everyone knows, all the factors mentioned above are affected by human's subjective intention. Thus, in these cases, it is impossible for us to predict the probability distributions of the returns of risky assets. To solve the problem of uncertainty, there are two routes. One is fuzzy theory. Seyed Jafar Sadjadi [8] considered several portfolio selection problems including probabilistic future returns with ambiguous expected returns assumed as random fuzzy variables. Yong etc. [9] deal with multi-period portfolio selection problems in fuzzy environment by considering some or all criteria, including return, transaction cost, risk and skewness of portfolio. Similar literatures see to [10-15]. Another is robust optimal. Robust optimization is another approach towards optimization under uncertainty. Anna [16] deal with a portfolio selection model in which the methodologies of robust optimization are used for the minimization of the conditional value at risk of a portfolio of shares. Yongma Moon [17] constructed the robust portfolio model represented portfolio risk by the return standard deviation, avoiding large computing problems. Seyed Jafar Sadjadi etc. [18] presented a new portfolio selection model for uncertain information, and solved the model with different robust method. Nalan [19] proposed the multi-period mean–variance portfolio optimization model with worst-case robust decisions. The mentioned literatures only pay attention to low frequency data, that is, the trading is controlled in day or even a month or longer time range. It is not reasonable. For example, there may exist a good chance in a day, also called intra-daily the investor should pursuit the most favourable business opportunities rather than continue to wait for the final trading deadline.

The rest of the paper is organized as follows. In Section 2, the multi-period mean variance optimization problem is described. In Section 3, we introduce the robust optimal methodology. Section 4 focuses on an empirical research of a optimal portfolio model with five risk assets in Chinese market. Conclusions are given in section 5.

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2 Mean- dynamic var multi-stage portfolio model

Suppose there are n alternative risk assets. $R_{it}, i=1,2\cdots n; t=1,2\cdots T$ are the yields of asset i at stage t, with expectation $r_{it}=E(R_{it})$ and covariance matrix $\Sigma_t=(\sigma_{ij}(t))_{n\times n}$, $\sigma_{ij}(t)=COV(R_{it},R_{jt})$. At the beginning of stage t, the portfolio share is x_{it}. And at stage t, buying and selling respectively are b_{it}, s_{it} .So, the portfolio share at stage t is $x_{i,t-1}+b_{it}-s_{it}$. The purchase and sale transactions need transaction costs, suppose the cost are $C(b_{it})$ and $C(s_{it})$. Another factor to consider is wealth constraint. Suppose an investor have the initial wealth S_0, then at stage t, its transition equation S_t: $S_t=(1+I_{pt})S_{t-1}$, where I_{pt} is the portfolio earnings, $I_{pt}=r_{pt}-\sum_{i=1}^{n}\left[C(b_{it})-C(s_{it})\right]$, in which the expected rate of return $r_{pt}=\sum_{i=1}^{n}r_{it}(x_{it}+b_{it}-s_{it})$.

At stage t, the VaR of the portfolio is $f_t=\Phi^{-1}(p)\sqrt{x_t^T\Sigma_t x_t}-r_t^T x_t$, $x_t=(x_{1t},x_{2t}\cdots x_{nt})^T$, $r_t=(r_{1t},r_{2t}\cdots r_{nt})^T$.

The optimal dynamic multi-stage portfolio of mean-dynamic VaR is $\min f_t$

$$s.t\begin{cases} S_t=\prod_{l=1}^{t}(1+I_{pl})S_0 \\ 0\le b_{it}\le \sum_{j=1,j\ne i}^{n} x_{jt} \\ 0\le s_{it}\le x_{it} \\ b_{it}s_{it}=0 \end{cases} \quad \text{(Model 1)}$$

Generally, invest strategy aims to the future stage that at stage t according to stage $t-1,t-2\cdots 1$, which are the given information. In model 1, the objective is to minimax the portfolio risk at stage t, and the first constraint means the state transition equation of wealth. The second constraint express not to short buy. In addition, not to short sell in the third constraint. Buying and selling at the same time is limited.

We should describe the relation of the state transition equation of wealth and the initial wealth.

$$\begin{aligned} S_t &= (1+I_{pt})S_{t-1} \\ &= (1+I_{pt})(1+I_{p,t-1})S_{t-2} \\ &= \cdots \\ &= \prod_{l=1}^{t}(1+I_{pl})S_0 \end{aligned}.$$

3 The classical portfolio model and its robust counterpart

A rational investor does not aim solely at maximizing the expected return of an investment, but also at minimizing its risk. The MV optimization problem was formulated as follows (M1):

(M1) $\min_X X\Sigma X'$

$$\text{subject to}\begin{cases} \sum x_i=1 & (1) \\ \sum E(r_i)x_i=r_p & (2) \\ 0\le x_i\le 1 & (3) \end{cases}$$

where x_i are portfolio weights, r_i is the rate of return of instrument i, and Σ is the covariance. The second constraint requires portfolio's expected return to be equal to a prescribed value r_p.

This model is under certain and exact environment, but in real market, the inputs are changing, history cannot replace future. We consider the uncertain set for return mean. we define r' is the estimation of real value r, the uncertainly set I as $I=\left\{r: r_i'-s_i\le r_i\le r_i'+s_i\right\}$ for mean. According to Anna [16], the robust counterpart: $\min\sum r_i x_i\ge r_p$ can be transferred to the following form:

$$\begin{cases} \sum r_i' x_i-\sum s_i m_i\ge r_p \\ m_i\ge x_i \\ s_i\ge 0 \end{cases}$$

The model handling process reflects the robust optimization, the worst best solution.

4 Empirical research

4.1 THE MODEL DESCRIBE

In our model, for simply, let the confidence level $c=97.5\%$, so $\Phi^{-1}(p)=1.96$. We suppose an investor wish pursuit no less than 7% profit. So the first constraint is reformed as $S_t\ge(1+8\%)S_0$ or $\prod_{l=1}^{t}(1+I_{pl})\ge 1.08$. Transaction costs function is $C(b_{it})=0.008b_{it}$ and $C(s_{it})=0.008s_{it}$. It reforms the model step as

(i) select the high frequency minute data, divide them in terms of a given minutes(such as 5 minutes)and get the investment stages n.

(ii) calculate the sample mean and covariance of each stock in every stage, we express them as r_{it}, Σ_{it} approximately. To avoid the risk effectively, we consider the robust optimization. It means if we make a strategic decision at one stage, we change the sample mean and covariance in an interval whose left is the minimax and right is the maximum of the stages before current stage.

(iii) solve the optimal portfolio model. The solutions show that at each stage, in order to seek the objective of minimax risk under the constraints, how to adjust the invest share.

For simple, given the original respective share are $\frac{1}{n}$. We special the stage is 2 and 3. $x_1 = (x_{11}, x_{21} \cdots x_{n1})$, $b_1 = (b_{11}, b_{21} \cdots b_{n1})$, $s_1 = (s_{11}, s_{21} \cdots s_{n1})$. The construction and solution steps are

$$\min \quad f_1 = 1.96\sqrt{x_1^T \Sigma_1 x_1} - r_1^T x_1 .$$

The constraints are

$$\begin{cases} S_1 = (1 + I_{p1})S_0 \geq 1.08S_0 \quad or \quad 1 + I_{p1} \geq 1.08 \\ 0 \leq b_{i1} \leq \sum_{j=1, j\neq i}^{n} x_{j1} \\ 0 \leq s_{i1} \leq x_{i1} \\ b_{i1}s_{i1} = 0 \\ r_{p1} = r_1(x_1 + b_1 - s_1)^T \\ I_{p1} = r_{p1} - 0.008b_1 I^T + 0.008 s_1 I^T \end{cases} ,$$

where $r_1 = (r_{11}, r_{21} \cdots r_{n1})$ is the return vector and Σ_1 is the covariance at stage 1 from the data, I^T is the transpose of n-dimensional unit vector. To solve this model, we obtain the adjustment strategy b_1 and s_1. So then, the portfolio shares are updated.

For stage 2, the objective is

$$\min \quad f_2 = 1.96\sqrt{x_2^T \Sigma_2' x_2} - r_2'^T x_1 .$$

The objective is not the same as the stage 1 case. Where Σ_2' is not certain but change from $\min\{\Sigma_1, \Sigma_2\}$ to $\max\{\Sigma_1, \Sigma_2\}$, the same as the uncertain return r_2' changes from $\min\{r_1, r_2\}$ to $\max\{r_1, r_2\}$. The constraints are similar only except the first one is $(1 + I_{p1})(1 + I_{p2}) \geq 1.08$.

For stage 3, we also obtain the corresponding optimal model.

4.2 EMPIRICAL STUDY

In this part, it focuses on an empirical research of a optimal portfolio model with five risk assets in Chinese market who show a good momentum in 2013. We choose the high frequency data in 5 minutes of five stocks NO 002583600694, 600089, 601166 and 600276 from 2012-01-04 09:01:00 to 2013-9-4 15:00:00,58320 data, which is from GATA database. However, for simply, we construct the optimal portfolio model only suppose a stage conclude five moths. That is, there is 5 stages. From solving the model, the dynamic portfolio is:

At stage 1, $a_{11} = 0.8034, a_{21} = a_{31} = 0, a_{41} = 0.164, a_{51} = 0$ and $s_{11} = s_{21} = s_{31} = 0,\ s_{41} = s_{51} = 0$.

At stage 2, $a_{12} = 0, a_{22} = 0.775, a_{32} = 0, a_{42} = 0.083, a_{52} = 0$ and $s_{21} = s_{22} = s_{32} = 0,\ s_{42} = 0, s_{52} = 0.561$.

At stage 3, $a_{31} = a_{32} = a_{33} = a_{34} = a_{35} = 0$ and $s_{31} = s_{32} = 0, s_{33} = 0.11, s_{34} = s_{35} = 0$.

At stage 4 not do any tradings.

At stage 5, $a_{51} = 0.381, a_{52} = 0.042, a_{53} = a_{54} = a_{55} = 0$ and $s_{51} = 0, s_{52} = 0,\ s_{53} = 0.0844, s_{54} = s_{35} = 0$

5 Conclusion

Dynamic portfolio is superior than a single static model. And in reality, the return and covariance change with some factors of future, it is not inappropriate to use the history features to describe the future case. However, for an investor, how to control risk is the first factor that he should consider when making an investment. This paper construct the dynamic portfolio model under robust counterpart, in which it focus minimax the risk under the constraints. It also considers the wealth budget and transaction costs, which is very important in dynamic investing. Because it is not a sensible stuff to trade frequently ignoring commission. At last, an empirical study choosing five stocks from Chinese market to test validity of the models, giving the strategies. In fact, the solution this paper mentioned can be extended to higher frequency in the data model, investment strategy curve (including the sale point in time and quantity) could provide investors better suggestions.

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