The optimal promised quality defect model for service guarantees

Wenlong Wang[1, 2]*, Xinmei Liu[1, 2], Xiaojie Zhang[1, 2]

[1]*School of Management, Xi'an Jiaotong University, 28 Xiannin Road, Xi'an Shaanxi, China*

[2]*The key lab of the Ministry of Education for Process Control & Efficiency Engineering, 28 Xiannin Road, Xian Shaanxi, China*

Received 6 October 2013, www.tsi.lv

Abstract

Service quality guarantee is an important tool for firms to boost demands, put up prices, and enhance profits. However, when promised quality defect is too high or low, the impact on the organization and the customer is usually negative. Therefore, determining the level of promised quality defect is of critical strategic and tactical importance in businesses. Yet, systematic quantitative methods aren't found to help managers determine promised quality defect. We propose a simple but powerful model in finding the optimal promised service quality defect. The model makes trade-offs between benefits and costs of service defect guarantees. Firstly, the decision of promised quality defect is analysed when service price is exogenous. We secondly investigated when service price is endogenous, how can a service provider make decisions on service price and promised quality defect simultaneously to maximize its profit. Thirdly, comprehensive analysis of how service providers promise the optimal quality defect from two aspects of demand and supply is given. Numerical analysis is conducted to illustrate the interactive effect of endogenous service price and affected service supply. In the end, we conclude the paper and suggest areas for future research. With only definitional changes, the model can be applied to other guarantee contexts.

Keywords: quality guarantees; promised quality defect; service providers; affected service supply

1 Introduction

Service quality has received considerable attention in the rapidly developing service economy. Primarily due to the intangibility in the process of service production and consumption, firms have tended to adopt certain kind of service quality guarantee policy as the differentiation strategy to attract customers. A service quality guarantee policy assures the customer that during the transaction, if the actual service quality defect exceeds the promised level, the firm will have to bear the cost of service failure, such as compensation to customers, service recovery, and loss of goodwill.

Several studies indicate that service guarantees are a signal of quality and that customers follow this signal to judge product quality [1-3]. Similarly, many researchers argue that service guarantees decrease the perceived risk of customers [4]. As a result, the demand for the service will be increased. A great number of companies, especially those that are service-oriented, adopt this strategy to provide service in accordance with the service quality guarantees that they make in advance. The service quality guarantee policy has efficiently promoted service demand, sales and reputation, and helped those firms win customers. However, when the promised quality defect exceeded, the service provider incurs a substantial cost. Thus, how to make quality guarantee policies to balance the potential profits and costs is an issue for companies.

According to ref [5], a typical service guarantee policy includes two elements: a meaningful promise of a certain service quality defect and a compensation or pay-out offer. The extant literatures [6-8] primarily focus on the compensation for quality defect, providing little insight on the promised quality defect. For example, comparative static analysis was employed to derive optimal decisions of service price and compensation cost for quality defect [6], leaving promised quality defect being ignored, while defect commitment for service quality is the foundation for compensation. Only when clear defect commitment is determined, can service providers compensate to customers for excessive quality defects. Thus, the promised quality defect has to be taken into account when providers make compensation in their guarantee policy.

In addition, the effect of quality guarantees on service demand is thoroughly analysed in extant literature, yet, without considering that on service supply [6, 9, 10]. Specifically, when the promised quality defect is quite low, providers will have to bear more risk on quality defect. In order to mitigate such risks, service output will be reduced inevitably, making it difficult to meet customers' service demand. Conversely, in high promised quality defect condition, less risk will lead providers to supply more service to customers, exceeding actual market demand. Therefore, when making service quality guarantee policies, service providers have to consider the

* ***Corresponding author*** e-mail: wwl.1986@stu.xjtu.edu.cn

impact of quality defect commitment on their supply capacities.

In a word, promised quality defect, as a part of the quality guarantee policy, has not been systematically studied in prior research. Thus, this paper tends to analyse the optimal decision of promised quality defect. In the next section, we present a review of the existing literature on service quality guarantees. Then, we explain the conceptual model of the problem, identifying and rationalizing service demand and cost of quality defect. In section 4 a model of service quality guarantee is developed and analysed. In this section the decision of promised quality defect is analysed when service price is exogenous firstly. This section secondly investigated when service price is endogenous, how can a service provider make decisions on service price and promised quality defect simultaneously to maximize its profit, namely the joint optimal decision on service price and promised quality defect. Thirdly, in a competition-intense market where the service price can be taken as fixed, comprehensive analysis of how service providers promise the optimal quality defect from two aspects of demand and supply is given. In other words, when making the optimal decision on promised quality defect, the service provider will in advance take into account the impact of promised quality defect on its service supply, which will influence its profit by its relative deficiency or excess to the demand. In section 5 numerical analysis is conducted to illustrate the interactive effect of endogenous service price and affected service supply. Finally, Section 7 concludes the paper and suggests areas for future research.

2 Literature review

Service quality guarantee is an extension of product warranties, and it primarily can reduce the risk perceived by customers [11]. “Service failure” will occur when the service fail to meet the promise provided in a service guarantee program and then service remedy for the customer is needed according to the guarantee [4, 12].Service quality guarantee issues have been continuously been studied in recent years. The adopted setting in these studies is a single enterprise that provides a service quality guarantee to customers, with various methods, including theoretical model analysis [13, 14], experimental design [15, 16], and industry investigation [17, 18].

For instance, ref [15] employed a before–after experimental design with a role-playing approach to investigate the impact of a service guarantee on an outstanding versus a good service provider in the hotel industry. This research indicates that an explicit service guarantee does not negatively affect the outstanding service provider, and the impact on the good service provider is more significant than that on the outstanding service provider. With a conceptual model, ref [19] empirically examined the effects of service guarantees. They found that service reliability is customers’ primary interest and coming to the second is the interest in compensation for service failures. Their findings provide support to the idea that including service process evidence can lead to significantly increased customers’ willingness to purchase from the service provider. Furthermore, when service process evidence is listed with detail in the service quality guarantee, the compensation is more persuasive. Ref [13] developed a framework of service guarantee strength, in which they posited that high service guarantee effort can improve service quality, customer satisfaction, and customer loyalty. Ref [6] generalized existing blanket delivery-time guarantee models by drawing on concepts from other field. They relaxed simplifying assumptions to provide a comprehensive and practical model, and found that pricing policies are less critical than previously thought when the payment made for late delivery is included as part of the delivery-time guarantee policy. Ref [14] proposed a resource allocation and pricing mechanism for a service system that is subject to a class-dependent quality of service (QoS) guarantee. They suggested that the pricing scheme with QoS guarantee depends on the scheduling policy implemented and is characteristically different from that without the QoS guarantee. Ref [18] also empirically found that the type of service guarantee can significantly influence customers’ perceived quality and perceived risk. Ref [10] studied the quality decisions of the functional logistics service provider (FLSP) and the logistics service integrator (LSI) with a service quality defect guarantee promised by the FLSP. The optimal quality defect guarantee of the FLSP and the optimal quality supervision effort of the LSI are presented fewer than three typical game modes: Nash game, Stackelberg game, and centralized decision. Ref [7] proposed a quantitative model, the Economic Pay-out Model for Service Guarantees (EPMSG), for determining the optimal pay-out level for the service industry. Based on ref [7], ref [8] took a service guarantee level into consideration to obtain the optimal pay-out. They considered a generic model to provide insights into the dynamic interaction between the service guarantee and optimal pay-out levels.

It can be seen that previous research mainly focused on the positive effect of a service quality guarantee policy, such as customers’ satisfaction and loyalty. Promised quality defect has occasionally been mentioned to some extent in modelling research. For example, ref [10] did not systematically analyse promised quality defect (e.g., how to make decisions on quality defect when service price is endogenous and when the decisions can affect service supply) although they noticed the issue of promised quality defect in quality decisions.

To date, there are few researches focusing on optimizing the promised quality defect. The most relevant researches are ref [6] and ref [8].However, ref [6] discussed the optimal quality guarantee policy from the demand perspective without considering the impact of promised quality defect on service supply capacity.

Although ref [8] included quality commitment in optimal compensation for the quality defect, they viewed the actual service quality as exogenous, ignoring the nature of randomness in service production and delivery. Thus, this paper aims to contribute in two ways. Firstly, the actual service quality defect is taken as a random variable in order to better describe the real business practice, and then we try to solve how to make joint optimal decision of service price and promised quality defect when service price is endogenous. Secondly, the impact of promised quality defect on supply is included in the model. In other words, the optimal promised quality defect is decided combining the influence of demand and supply.

3 The conceptual model

Consider the situation where a service provider that has adopted a quality guarantee policy supplies service to the market. The service provider guarantees a quality defect level q. On one hand, when the actual quality defect X is greater than q, the provider will have to bear the defect cost, including the compensation for the customer and the reputation loss resulting from higher quality defect than its promised level. On the other hand, promised quality defect will also enhance service demand by attracting more customers through less risk perceived by customers. In brief, how to balance the revenue increased from boomed demand and cost from taking the risk of quality defect when designing a service quality guarantee policy is the primary concern for the service provider.

3.1 SERVICE DEMAND

Ref [20] used the exponential function to depict the relationship between service demand and quality guarantees. Ref [10] also adopted the exponential function when studying the quality guarantee policy in supply chain. However, their service demand function only includes promised quality defect while ignored the effect of service price, which is one of the main factors when customers purchase services, on service demand. Therefore, besides promised quality defect, this paper also considers the impact of price on service demand. The function of service demand expresses as $D(p,q)=\eta e^{-\varepsilon p-wq}$, where η signifies the total service demand, p is the service price, and ε and w are elasticity of service price and promised quality defect respectively.

3.2 COST OF QUALITY DEFECT

The production and delivery of service is randomly influenced by some factors such as adverse weather and mistakes of front-line service staff. Consequently, service received by customers is virtually unreliable. The quality defect cost will be incurred if the actual service quality defect X exceeds the promised quality defect q.

Explicit and implicit costs are supposed to be both included in the cost of quality defect. The explicit cost mainly refers to the payment to customers when the actual quality defect exceeds q. Of course, if there is a specific payment that is being made, these are appropriate measures. In addition to those payments, unrecorded or “hidden” quality costs such as customer dissatisfaction and loss due to bad reputation should also be included as part of the defect cost; these types of costs are generally not part of current accounting systems [21, 22] and must be incorporated separately. Also, a firm may make a quality promise without a guaranteed monetary payment; still, customer dissatisfaction is an indirect cost if the promise is not met. However, since the implicit quality defect cost is difficult to measure and obtain, we merely focus on the explicit one similar to the studies of ref [6].

The cost of exceeding guaranteed quality defect has previously been modelled in two ways: (1) using a fixed payment to the customer regardless of how severe the quality defect is, and (2) using a payment that is a function of the degree of quality defect, which is the difference between the actual quality defect and promised quality defect. In general, the latter way, namely variable defect cost, can not only help service providers to improve their service, but also better remedy the reputation loss by compensating customers who suffered from service quality defect. Given this, the quality defect cost in this paper is in accordance with the variable quality defect cost. The product quality defect literature has embraced the well-known quadratic loss function as an appropriate measure of the second type of quality defect cost. Ref [23] analysed the rationality of the adoption of quadratic function when the actual quality defect exceeds the promised quality defect. Thus, based on the work of ref [23], the function of quality defect cost in this paper is $C(q)=c\int_q^{\infty}(X-q)^2 f(X)dX$, where $(X-q)$ signifies the degree of quality defect, $f(X)$ is its probability density function, and c is the unit cost for quality defect.

Without loss of generality, it is assumed that there is no fixed production cost and the variable cost per unit is v ($v<p$). The profit function of a service provider is as following.

$$\Pi(p,q)=D(p,q)\left[p-v-C(q)\right]$$
$$=\eta e^{-\varepsilon p-wq}\left[p-v-c\int_q^{\infty}(X-q)^2 f(X)dX\right] \quad (1)$$

4 The analytic model

4.1 WITH EXDOGENENOUS SERVICE PRICE

We begin by analysing the promised quality defect q of a service provider in this section, where q_b^* signifies the optimal promised quality defect, and subscript $_b$ is the benchmark. Suppose that service price p is exogenous. The service provider tries to maximize its profit by promising the level of quality defect q.

Consider $\Pi_b(q)$ signifies the profit function of the service provider. As p is constant, the optimal promised quality defect q_b^* can be derived from the first order condition (FOC) of $\Pi_b(q)$. Then, the concavity of $\Pi_b(q)$ can be obtained from the second order condition (SOC) of $\Pi_b(q)$. The function expression of $f(X)$ is needed for FOC and SOC. Drawn on the work of ref [10], assume that X is a random variable with the exponential distribution, and the mean is $\frac{1}{\lambda}$.

Proposition 1 There exists one and only one optimal q_b^* that maximizes the service provider's profit function $\Pi_b(q)$. Specifically, $\Pi_b(q)$ increases concavely in $\left(0,q_b^*\right)$, decreases concavely in $\left[q_b^*,q_b^{'}\right]$, and decreases convexly in $\left(q_b^{'},\infty\right)$, where $q_b^*=-\frac{1}{\lambda}\text{In}\frac{(p-v)w\lambda^2}{2c(w+\lambda)}$, $q_b^{'}=-\frac{1}{\lambda}\text{In}\frac{(p-v)w^2\lambda^2}{2c(w+\lambda)^2}$.

Proof in Appendix.

From Proposition 1, it can be seen that when service price p is exogenous, there is an optimal promised quality defect for the service provider. To be specific, $\Pi_b(q)$ is concave-convex in q, with the inflection point of $q_b^{'}$, and the unique optimal value is q_b^*. That is to say, when faced with fixed service price p, the provider can maximize its profit by promising quality defect q_b^*.

There are two effects of the promised quality defect on the profit of the service provider. The first is called commitment-demand effect, which is the negative effect of the promised quality defect on the provider's profit via service demand. The second one is commitment-marginal-profit effect, referring to the positive effect of promised quality defect on provider's profit through the marginal profit of the service. In $\left(0,q_b^*\right)$, the commitment-demand effect is weaker than the commitment-marginal-profit effect, leading to a continually increased service profit. However, the commitment-demand effect becomes stronger than the commitment-marginal-profit effect in $\left[q_b^*,+\infty\right)$. Thus, the service profit in $\left[q_b^*,+\infty\right)$ is decreasing. At the critical point q_b^* where the two kinds of effects reach a balance, the service provider can maximize its profit.

4.2 WITH ENDOGENENOUS SERVICE PRICE

Service price is assumed to be endogenous in this section. Joint optimal decisions on service price and promised quality defect need to be made. In other words, optimal service price p_p^* and optimal promised quality defect q_p^* to maximize the provider's profit are derived simultaneously, where the subscript p signifies endogenous service price. Since the provider's profit function $\Pi_p(p,q)$ is not jointly concave in p and q, it is impossible to find the optimal joint decision by negative definite Hessian Matrix. But the two-stage optimization method can solve the joint decision problem of service price and promised quality defect [24, 25]. In the first stage, promised quality defect is assumed to be constant, and then the optimal service price function of promised quality defect $p^*(q)$ can be obtained through the FOC of $\Pi_p(p,q)$ to service price. In stage 2, substituting $p^*(q)$ into the original profit function $\Pi_p(p,q)$, we can derive a new profit function $\Pi_p(p^*(q),q)$. If the new profit function $\Pi_p(p^*(q),q)$ has a maximum in q, then the original profit function $\Pi_p(p,q)$ can also be maximized in p and q, which is the optimal joint decision on service price p and promised quality defect q.

4.2.1 The optimal service price function of promised quality defect

In this section, the optimal service price p^* is obtained when promised quality defect q is given. It means that when certain quality guarantee policy is adopted, quality defect can be considered as exogenous. Thus, the problem for the service provider is how to pricing the service to maximize its profit.

As promised quality defect is exogenous, the profit function of the service provider is $\Pi_p(p)$. The optimal service price can be obtained through the FOC of $\Pi_p(p)$. Then the SOC will show concavity of the profit function $\Pi_p(p)$.

Lemma 1 Given the promised quality defect q, the unique optimal service price is p^*, which can maximize the provider's profit function $\Pi_p(p)$. Specifically, the provider's profit function $\Pi_p(p)$ increases in $\left(0,p^*\right)$, decreases convexly in $\left[p^*,p'\right]$, and decreases concavely in $\left(p',\infty\right)$, where $p^*=\frac{1}{\varepsilon}+v+c\int_q^\infty (X-q)^2 f(X)dX$, $p'=\frac{2}{\varepsilon}+v+c\int_q^\infty (X-q)^2 f(X)dX$.

Proof in Appendix.

Lemma 1 shows that when the promised quality defect is given, there exists an optimal pricing policy. To be specific, the service provider's profit function $\Pi_p(p)$ is convex-concave in the service price p with the inflection point of p', and the unique maximum value is p^*. That is to say, when faced with fixed promised quality defect, the service provider can maximize its profit by pricing at p^*.

As for the service price, there are also two kinds of effects on the service profit. One can be called price-demand effect, which is the negative effect of the service price on provider's profit via service demand. The other is price-marginal-profit effect, referring to the positive effect of the service price on the provider's profit through the marginal profit of the service. In $\left(0, p^*\right)$, the price-demand effect is weaker than the price-marginal-profit effect, leading to a continually increased service profit. However, in $\left[p^*, +\infty\right)$, the price-demand effect becomes stronger than the price-marginal-profit effect. Thus, the service profit in $\left[p^*, +\infty\right)$ is decreasing. At the critical point p^* where the two effects reach a balance, the service provider can maximize its profit.

4.2.2 The optimal quality defect for profit function

Substituting $p^* = \frac{1}{\varepsilon} + v + c\int_q^\infty (X-q)^2 f(X)dX$ into the service provider's original profit function $\Pi_p(p,q)$, the new profit function is expressed as $\Pi_p(p^*(q),q)$. The monotonicity and concavity of the profit function $\Pi_p(p^*(q),q)$ in promised quality defect q_p can be obtained from the FOC and SOC. Consequently, the optimal promised quality defect q_p^* can be derived.

Lemma 2 when $p^* = \frac{1}{\varepsilon} + v + c\int_q^\infty (X-q)^2 f(X)dX$, there exists one and only one optimal promise quality defect q_p^* that can maximize the provider's profit function $\Pi_p(p^*(q),q)$. When $\frac{1}{\lambda} \geq \frac{w}{2c\varepsilon}$, $q_p^* = \frac{1}{\lambda}\text{In}\frac{2c\varepsilon}{w\lambda}$ and when $\frac{1}{\lambda} < \frac{w}{2c\varepsilon}$, $q_p^* = 0$.

Proof in Appendix.

Although the provider's profit function $\Pi_p(p^*(q),q)$ is not concave in promised quality defect, there always exists one and only one maximum value q_p^* by analysing the monotonicity of the profit function. Since $\frac{1}{\lambda}$ is the expected value of actual service quality, the provider can make optimal promised quality defect according to its actual service quality. Specifically, increased demand brought by decreased promised quality defect, will have a positive effect on the provider's profit at an increasing rate; however, increased cost from less quality defect will have a negative effect on the provider's profit at an increasing rate. The net effect determines how the provider will act. When $\frac{1}{\lambda} < \frac{w}{2c\varepsilon}$, the negative effect of cost on profit is far weaker than the positive effect of demand on profit due to the provider's high qualified service. Thus, if the actual quality defect is at a low level, the policy of promised quality defect is beneficial for the provider's profit. That is to say, promising zero quality defect is the optimal choice for the provider in this condition. However, when $\frac{1}{\lambda} \geq \frac{w}{2c\varepsilon}$, the negative effect of cost on profit is much stronger than the positive effect of demand on profit due to the provider's poor service quality. Hence, if the actual quality defect increased to a high level, the effect of promised quality defect on the provider's profit is changing from positive to negative. It means that promising appropriate quality defect is the optimal choice for the provider in this condition.

Based on the two-stage optimization method, Proposition 2 is obtained combining Lemma 1 and Lemma 2.

Proposition 2 There exists one and only one joint optimal p and q that can maximize the provider's profit function $\Pi_p(p,q)$, where the optimal service price is $p^* = \frac{1}{\varepsilon} + v + \frac{w}{\lambda\varepsilon}$ and the optimal promised quality defect is $q_p^* = -\frac{1}{\lambda}\text{In}\frac{w\lambda}{2c\varepsilon}$.

Proposition 2 demonstrates that although the provider's profit function $\Pi_p(p,q)$ is not jointly concave in p and q, the two-stage optimization method can help to solve the joint decision problem of service price p, and promised quality defect q. Proposition 2 also exhibits that quality guarantee is a two-dimensional strategy. Service price and promised quality defect both need to be taken into consideration when the provider adopts quality guarantee as a differentiation strategy to compete in the market. Neither the service price nor the promised quality defect alone can maximize the provider' profit. Making quality guarantee maybe incur increased cost for quality defect to some extent, but also can boost demand from market due to promised quality defect. Thus, the provider can realize its maximized profit through joint optimal decision of service price and promised quality defect.

To make it more visualized, numerical analysis by MAPLE 17 software to verify the validity of proposition 2 is shown in Figure 1. Assume that $\eta = 20000$, $\varepsilon = 0.8$, $w = 0.2$, $c = 0.5$, $v = 1$, $\lambda = 1$.

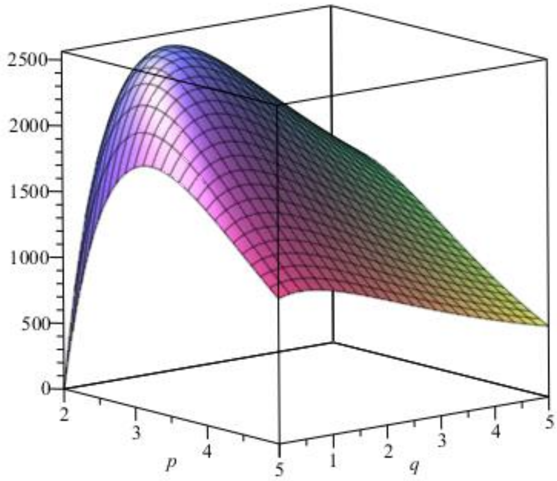

FIGURE 1 Profit function of the service provider on the service price p and promised quality defect q

From Figure 1, it can be seen that the profit function $\Pi_p(p,q)$ is jointly quasi-concave on the service price p and promised quality defect q. There is a unique $p^*=2.5$ and unique $q_p^*=1.386$ that can maximize the provider's profit, which is $\Pi_p(p^*,q^*)=2564$.

4.3 WITH AFFECTED SERVICE SUPPLY

Promised quality defect can affect the service supply for the following two reasons, occupancy of resources for service production and the avoidance of quality risks. Firstly, when the promised quality defect is at a low level, part of resources will be used to improve and maintain high service quality, such as training programs for front-line employees and procedure improvement; otherwise, these resources should have been used to expand the production scale. Secondly, the lower the promised quality defect is, the more the risk is inherent. From the view of intrinsic preference to mitigate risks, the provider will supply less service to the market. Thus, the supply capacity of the provider is more limited when the service is of low promised quality defect than when that of high promised quality defect. The supply S increases monotonically with q, $\frac{\partial S}{\partial q}>0$, at an increasing rate, $\frac{\partial^2 S}{\partial q^2}>0$ based on theory of increasing marginal cost. For the purpose of simplicity and consistency (with the early mentioned form of service demand function), the supply function is expressed as $S(q)=\xi e^{\mu q}$, where ξ is the total service supply of the provider, $e^{\mu q}$ is the proportion of service volume to the provider's total supply, and μ is the elasticity of the provider supply to the quality defect guarantee. On this occasion, the supply does not necessarily have to satisfy the demand from the market. The effective supply is influenced by the provider's promised quality defect. On one hand, although the service with low promised defect quality is very much in demand, the actual supply is quite small due to the high requirement of the service for the provider. It means that the effective supply is the actual supply regardless of the demand. On the other hand, however, the supply of high promised defect quality will be great while the demand is rather small. That is to say, the supply that exceeds the demand is meaningless for customers. In this case, the effective supply is the demand. Thus, the effective supply can be expressed as $\min(D(q),S(q))$. Drawn on the model in Section 3, the provider's profit function can be expressed as $\Pi_S(q)=\min(D(q),S(q))[p-v-C(q)]$, where the subscript S represents the effective service supply.

The demarcation point $q^{\#}=\frac{In(\frac{\eta}{\xi})-\varepsilon p}{\mu+w}$ can be derived from $D(q)=S(q)$. When the promised quality defect is less than the demarcation point, $q\le q^{\#}$, service supply is less than its demand. The new profit function of the service provider now is $\Pi_S(q)=S(q)[p-v-C(q)]$, called the supply profit function. Otherwise, the new profit function is $\Pi_S(q)=D(q)[p-v-C(q)]$, called the demand profit function , when $q>q^{\#}$.

Thus, the optimal promised quality defect will be identified by whether the supply is greater than the demand. Local optimum is firstly derived in order to obtain the global optimal in the final step.

4.3.1 Supply is less than demand

The derivation analysis of the provider's profit function is used in this section to figure out the local optimal decision in the condition of $q\le q^{\#}$.

Lemma 3 When the supply is less than the demand, $q\le q^{\#}$, there is an optimum in $[0,q^{\#}]$, which is $q_S^*=q^{\#}$, that can maximized the provider's profit function $\Pi_S(q)$. Specifically, If the actual quality defect is low ($\frac{1}{\lambda}\le\frac{1}{\mu}$), the provider's profit function monotonically increases with promised quality defect. If the actual quality defect is high ($\frac{1}{\lambda}\le\frac{1}{\mu}$), the provider's profit first decreases and then increases with the increase of promised quality defect.

Lemma 3 shows the condition where the service supply is less than the demand caused by the low level promised quality defect made by the service provider. If the actual quality defect is low ($\frac{1}{\lambda} \le \frac{1}{\mu}$), the provider's profit function monotonically increases with promised quality defect. In this case, the marginal profit $p-v-C(q)$ is positive because the quality defect cost of the provider $C(q)$ is quite small when the actual quality defect is at a low level. Therefore, the service provider will continually enlarge its supply because the marginal profit increases with the promised quality defect. If the actual quality defect is high ($\frac{1}{\lambda} \le \frac{1}{\mu}$), the provider's profit first decreases and then increases with the increase of promised quality defect. The reason is that When the promised quality defect is rather low ($q < q_s^{'}$, and $q_s^{'} = \frac{1}{\lambda} In \frac{2c(\mu-\lambda)}{\mu\lambda^2(p-v)}$), the quality defect cost for the provider $C(q)$ is rather great, or even greater than the marginal revenue $p-v$, resulting in a negative marginal profit . With the increase of the promised quality defect, the supply $S(q)$ increases while the marginal profit decreases. That is to say, the more the service provider supplies to the market, the more it will lose. When the promised quality defect is rather high ($q \ge q^c$), the quality defect cost of the provider is far less than the marginal revenue, resulting in a positive marginal profit. With the increase of the promised quality defect, the supply and the marginal revenue both increase. Thus, the provider's profit increases with promised quality defect.

4.3.2 Supply is more than demand

When the supply is greater than the demand, the provider's profit function is $\Pi_S(q) = D(q)[p-v-C(q)]$. From Proposition 1, it can be seen that the provider maximizes its profit when $q_b^* = -\frac{1}{\lambda}\text{In}\left(\frac{pw\lambda^2}{2c(w+\lambda)}\right)$. As $q > q^{\#}$, the relationship of magnitude between q_b^* and $q^{\#}$ will impact the optimal decision of the promised quality defect q . From Proposition 1, Lemma 4 is obtained.

Lemma 4 When the supply is greater than the demand, $q > q^{\#}$, consider two cases: if $q_b^* > q^{\#}$, the optimal promised quality defect is obtained when $q_S^* = q_b^*$; otherwise, the optimal solution is $q_S^* = q^{\#}$.

The global optimal decision can be obtained by synthetically analysing the local optimal in the two above mentioned parts.

Proposition 3 When the supply is affected by the promised quality defect, consider two cases: if $q_b^* > q^{\#}$, the optimal promised quality defect is obtained when $q_S^* = q_b^*$; otherwise, the optimal solution is $q_S^* = q^{\#}$.

Proposition 1 in section 4.1 indicates that if, regardless of the supply, the quality defect can only affect the demand, there is an unique optimal service quality defect. However, Proposition 3 also demonstrates that taking the supply and the demand simultaneously into account, there are two cases for the provider's optimal decision on service quality defect. Specifically, if $q_b^* > q^{\#}$, the provider's profit function, $\Pi_S(q)$, increases convexly in $[0, q^{\#}]$, increases concavely in $(q^{\#}, q_S^*]$ and decreases concavely in $(q_S^*, +\infty)$. Thus, the provider can maximize its profit when $q = q_S^*$. Intuitively, when $q_b^* > q^{\#}$, the intersection of the supply profit function curve and the demand profit function curve is to the left of the maximum of the original demand profit function, which is not included in the impacted area of the provider's profit function from the supply (the actual supply profit function). In addition, in most area affected by the supply, the provider's profit is less than that when there is no affect from the supply. Thus, the optimal of the original demand profit function is the same as that with simultaneous influence from the supply and the demand (see figure 2). If $q_b^* \le q^{\#}$, the provider's profit function, $\Pi_S(q)$, increases convexly in $[0, q^{\#}]$, and decreases concavely in $(q^{\#}, +\infty)$. Thus, the provider can maximize its profit when $q = q^{\#}$. Intuitively, when $q_b^* \le q^{\#}$, the intersection of the supply profit function curve and the demand profit function curve is to the right of the maximum of the original demand profit function, which is included in the impacted area of the provider's profit function from the supply), leading to the difference between the optimal of the original demand profit function and that with simultaneous influence from the supply and the demand. Besides, the local optimal in the area affected by supply becomes the global optimal in this condition (see Figure 3).

To make it more visualized, numerical analysis by MAPLE 17 software to verify the validity of proposition 3 is shown in Figure 2 and Figure 3. Assume that $\eta = 3000000$, $\varepsilon = 0.8$, $w = 0.6$, $c = 2.5$, $v = 1$, $\lambda = 0.5$, $\mu = 1.2$, $\beta = 0.5$, $p = 4$, $\xi = 30$ or $\xi = 10$.

Figure 2 and Figure 3 provide support for Proposition 3. Figure 2 shows that if $q_b^* > q^{\#}$, the local optimal solution when the supply is more than the demand is better than that when the supply is less than the demand. Thus, the service provider can maximize its profit at $q_S^* = q_b^*$. The unique optimal promised quality defect is $q_S^* = 5.007$ and the maximized profit is $\Pi_S^*(q_b^*) = 8182.357$. Otherwise, as indicated in Figure 3, if $q_b^* \le q^\#$, the local optimal solution when the supply is less than the demand is better than that when the supply is more than the demand. Thus, the service provider can maximize its profit at $q_S^* = q^\#$. The unique optimal promised quality defect is $q_p^* = 5.223$ and the maximized profit is $\Pi_S^*(q_b^*) = 8070.553$.

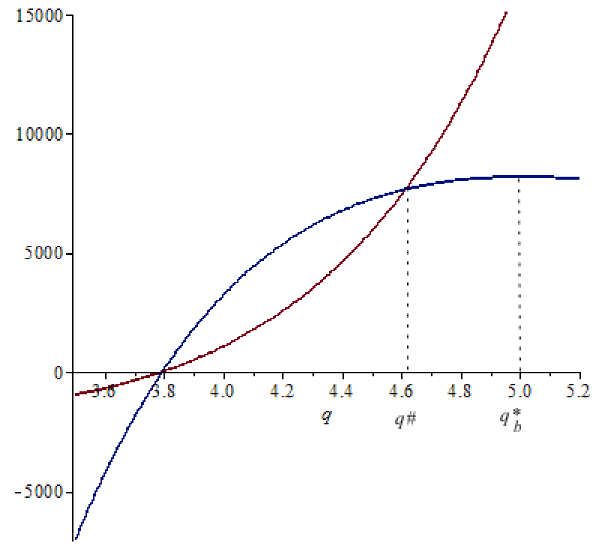

FIGURE 2 The provider's profit function of promised quality defect when $q_b^* > q^\#$

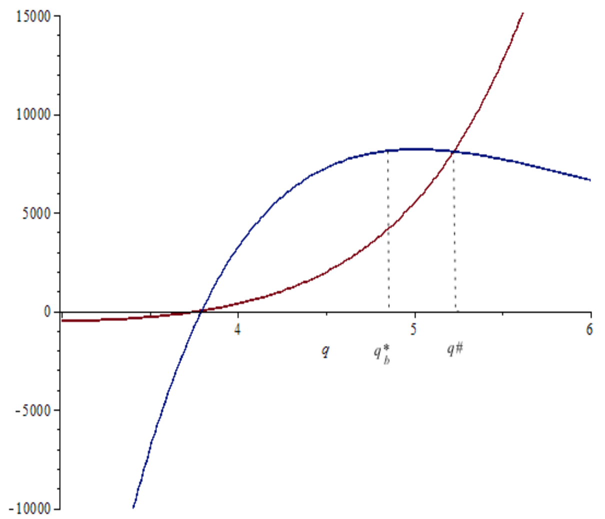

FIGURE 3 The provider's profit function of promised quality defect when $q_b^* \le q^\#$

5 Numerical analysis

Section 4 provided the quantitative analysis of the optimal promised quality defect when service price is endogenous and when service supply is affected by the quality guarantee policy, respectively. At the same time, the case with the interaction of endogenous service price and affected service supply is also taken into consideration. However, the closed form solution cannot be mathematically derived due to the complexity of the model. Thus, the numerical analysis is used to obtain the joint optimal decision of service price and promised quality defect when the supply is affected by the service guarantee policy.

In the similar vein with section 4.3, the supply function of the service provider is $S(p,q) = \xi e^{\mu q + hp}$, where h represents the sensitivity of supply to service price. Then, the effective supply is $\min(D(p,q), S(p,q))$. The profit function of the provider can be acquired based on the model in Section 3 as $\Pi_S(p,q) = \min(D(p,q), S(p,q))[p - v - C(q)]$ .Reasonable assignment is chosen ($\eta = 30000$, $\varepsilon = 0.8$, $w = 0.2$, $c = 0.5$, $v = 1$, $\lambda = 1$, $\xi = 5$) for the numerical analysis in order to intuitively get the joint optimal decision of service price and promised quality defect. As the sensitivity of the supply function significantly influences the joint optimal decision, the low and the high sensitivity of the supply function are considered separately.

5.1 LOW SENSITIVITY OF THE SUPPLY FUNCTION

When the sensitivity of the supply function to the service price and promised quality defect is low ($\mu = 0.6$, $h = 0.8$), the profit function $\Pi_S(p,q)$ on p and q can be drawn as following.

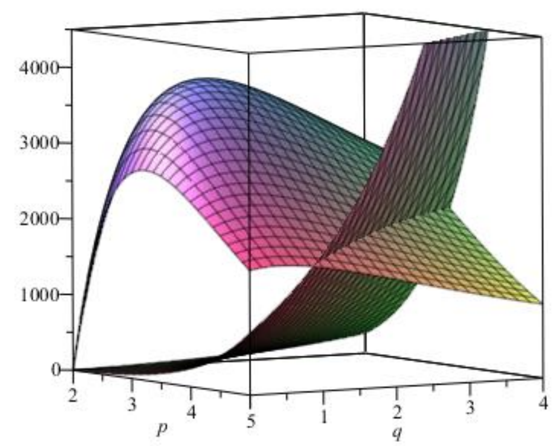

FIGURE 4 Overall profit function with low sensitivity

Figure 4 shows that on account of the affected supply, the joint optimal decision of service price and promised quality defect is codetermined by the supply profit surface and the demand profit surface. In the case where the intersection curve of the two surfaces lies in the outside of the optimal point of the demand profit surface, the optimal point is on the intersection curve. Further analysis of the intersection curve indicates that the FOC solution of profit function on p and q at the intersection curve gives the joint optimal decision of service price and promised quality defect, $p^* = 3.4276$ and $q^* = 4.0127$. Thus, the maximized profit of the service provider is $\Pi_S(p^*, q^*) = 2076.2$.

As a result of the low sensitivity of the supply function to service price and promised quality defect, the supply rises slowly with the increase of the service price and promised quality defect, leading to an unsatisfied demand, which means that the optimum has not reached the extreme point of the demand profit surface. Only when the service demand is met, meaning on the intersection curve, the profit of the provider can be maximized.

5.2 HIGH SENSITIVITY OF THE SUPPLY FUNCTION

When the sensitivity of the supply function to service price and promised quality defect is high ($\mu = 2.5$, $h = 2.5$), the profit function $\Pi_S(p, q)$ on p and q is shown as following.

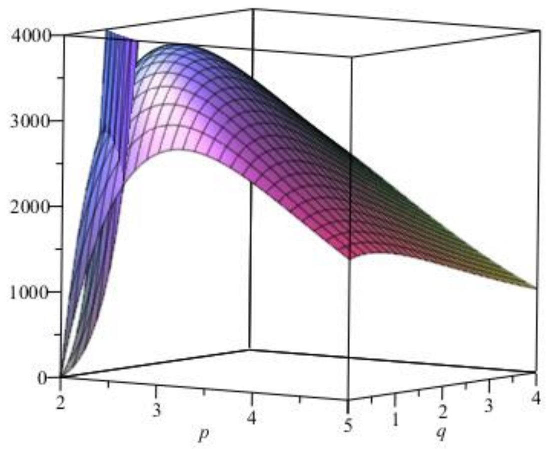

FIGURE 5 Overall profit function with high sensitivity

Figure 5 demonstrates that in comparison with the condition where there is no affected supply, the profit function of the provider has changed in this condition, yet owing to the much less vulnerability to the affected supply, the optimal point is on the demand profit surface. Although the overall profit surface changed, the intersection curve of the demand profit surface and supply profit surface is located at the inside of the optimal point of the demand profit surface. Thus, the changed part of the supply surface is still below the extreme point of the unchanged part of the demand surface, meaning that the optimum on the demand profit surface is the point that maximizes the overall profit function of the provider in this condition. The joint optimal decision is $p^* = 2.5$ and $q^* = 1.3863$. Thus, the maximized profit of the service provider is $\Pi(p^*, q^*) = 3825.2237$.

Due to the high sensitivity of the supply function to the service price and promised quality defect, the supply raises rather quickly with the increase of the service price and promised quality defect, leading to an effective satisfied demand. Therefore, the optimum is the extreme point of the demand profit surface.

Combing the analysis in 5.1 and 5.2, it is known that when the effect of service price and promised quality defect on the supply function is quite low, the joint optimal decision of the provider is on and can be derived from the intersection curve of the supply profit surface and the demand profit surface; however, when the effect of service price and promised quality defect on the supply function is quite high, the joint optimal decision of the provider depends on the extreme point of the demand profit surface. In other words, the joint optimal decision of the provider can be obtained from the FOC of the demand profit function on the service price and promised quality defect.

6 Conclusions

Service quality guarantee is an important tool for firms to boost demands, put up prices, and enhance profits. This paper presents a simple but powerful model in finding the optimal promised service quality defect. The model makes trade-offs between benefits and costs of service defect guarantee. With only definitional changes, the model can be applied to other guarantee contexts in which the demand and supply are influenced by service guarantees and actual service defect variable follows the exponential distribution.

We adopt an analytical approach for optimal quality defect promise of a firm making quality guarantees on their service. The proposed model generalizes existing service quality guarantee models in two primary aspects:

(1) The existing literature concerning service quality guarantees mainly focuses on service price and payment made for defect. The proposed model also includes the promised quality defect as a main decision variable and incorporates it as a part of the demand function. Further, the joint decision of price and promised quality defect is discussed when the service price is endogenous.

(2) Previous studies on service quality guarantees neglect the impact of promised quality defect on service supply capacity, which is included in the proposed model to further analyse its influence on provider's profit. Consequently, the optimal decision of promised quality defect is derived in this condition. This finding will better guide service providers to make the quality guarantee policy.

In addition, numerical analysis provides an intuitive joint optimal decision on service price and promised quality defect when the supply is vulnerable to the latter. When the sensitivity of supply function is high, the affected supply can hardly change the overall profit surface of the service provider. In this case, the joint optimal decision is the extreme point of the demand profit surface, which is consistent with joint optimal decision in 4.2 where the affected supply has no effect. However, when the sensitivity of supply function is low, the overall profit function is, to a great extent, impacted by the affected supply. Then the joint optimal decision is on the intersection curve of the demand profit surface and the supply profit surface, which is inferior to the condition in 4.2 where the affected supply has no effect.

An interesting direction of future research would be how a service provider can make promised quality defect to differentiate itself from its rivals in a competitive market. Moreover, what is the effect of industrial characteristics on service quality guarantee in some special service industry, either quite new or with greater risks (e.g. finance and information security) is merely studied. Last but not the least, from the perspective of supply chain, it is also intriguing that how the promised quality defect of a service provider can affect decisions of upstream and downstream members, the profit of the whole service supply chain, and further the coordination mechanism of the service supply chain based on quality guarantee.

Appendix

Proof of proposition 1

Because X is a random variable with the exponential distribution and the mean is $1/\lambda$, the service provider's profit function is

$$\begin{aligned}\Pi(q) &= \eta e^{-\varepsilon p - wq}\left[p - v - c\int_q^\infty (X-q)^2 f(X)dX\right] \\ &= \eta e^{-\varepsilon p - wq}\left[p - v - \frac{2ce^{-\lambda q}}{\lambda^2}\right]\end{aligned}. \quad (2)$$

Then, according to the first-order condition of the service provider's profit function, we can obtain that

$$q_b^* = -\frac{1}{\lambda}\text{In}\frac{(p-v)w\lambda^2}{2c(w+\lambda)}. \quad (3)$$

Then, when $q \le q_b^*$, $\frac{d\Pi(q)}{dq} \ge 0$; and when $q > q_b^*$, $\frac{d\Pi(q)}{dq} < 0$. So there exists a unique optimal promised quality defect q_b^*, which can maximize the profit of the service provider.

According to the second-order condition of the service provider's profit function, we can obtain that

$$q_b^{'} = -\frac{1}{\lambda}\text{In}\frac{(p-v)w^2\lambda^2}{2c(w+\lambda)^2}. \quad (4)$$

Then, we have that

$$\begin{cases}\frac{d^2\Pi(q)}{dq^2} \le 0, & if\ q \le q_b^{'} \\ \frac{d^2\Pi(q)}{dq^2} > 0, & if\ q > q_b^{'}\end{cases}. \quad (5)$$

Since $\frac{w}{w+\lambda} < 1$, $\text{In}\frac{w}{w+\lambda} < 0$, we can obtain that

$$q_b^{'} - q_b^* = -\frac{1}{\lambda}\text{In}\frac{w}{w+\lambda} > 0. \quad (6)$$

$\Pi_b(q)$ increases concavely in $(0, q_b^*)$, decreases concavely in $[q_b^*, q_b^{'}]$, and decreases convexly in $(q_b^{'}, \infty)$.

Proof of lemma 1

Since the service provider's profit function is

$$\begin{aligned}\Pi_p(p) &= D(p)[p - v - C(q)] \\ &= \eta e^{-\varepsilon p - wq}\left[p - v - c\int_q^\infty (X-q)^2 f(X)dX\right]\end{aligned}. \quad (7)$$

Then, according to the first-order condition of the service provider's profit function, we can obtain that

$$p^* = \frac{1}{\varepsilon} + v + c\int_q^\infty (X-q)^2 f(X)dX. \quad (8)$$

Since $\begin{cases}\frac{d\Pi_p(p)}{dp} \ge 0, & if\ p \le p^* \\ \frac{d\Pi_p(p)}{dp} < 0, & if\ p > p^*\end{cases}$, (9)

there exists a unique optimal service price p^* that can maximize the profit of the service provider.

$$\frac{d^2\Pi_p(p)}{dp^2} = \left[-2 + \varepsilon\left(p - v - c\int_q^\infty (X-q)^2 f(X)dX\right)\right]\eta e^{-\varepsilon p - wq}. \quad (10)$$

According to the second-order condition of $\Pi_p(p)$, we can obtain that

$$p' = \frac{2}{\varepsilon} + v + c\int_q^\infty (X-q)^2 f(X)dX. \quad (11)$$

Besides, $p' - p^* = \frac{1}{\varepsilon} > 0$.

So, the provider's profit function $\Pi_p(p)$ increases convexly in $\left(0,p^*\right)$, decreases convexly in $\left[p^*,p'\right]$, and decreases concavely in $\left(p',\infty\right)$.

Proof of lemma 2

Because X is a random variable with the exponential distribution and the mean is $1/\lambda$, the optimal service price is $p^*=\frac{1}{\varepsilon}+v+\frac{2ce^{-\lambda q}}{\lambda^2}$.

Then, we can obtain that

$$\Pi(p^*(q),q)=\frac{\eta}{\varepsilon}e^{-1-\varepsilon v-\frac{2c\varepsilon e^{-\lambda q}}{\lambda^2}-wq}. \quad (12)$$

The first derivative of service provider's profit function $\Pi(p^*(q),q)$ is

$$\frac{d\Pi(p^*(q),q)}{dq}=-\frac{\eta}{\varepsilon}e^{-1-\varepsilon v}\left(w-\frac{2c\varepsilon}{\lambda}e^{-\lambda q}\right)e^{-\frac{2c\varepsilon e^{-\lambda q}}{\lambda^2}-wq}. \quad (13)$$

According to the first-order condition of $\Pi(p^*(q),q)$, we can obtain that $q^*=-\frac{1}{\lambda}\text{In}\frac{w\lambda}{2c\varepsilon}$.

$$\begin{cases}\frac{d\Pi(p^*(q),q)}{dq}\geq 0, & if\ q\leq q^*\\ \frac{d\Pi(p^*(q),q)}{dq}<0, & if\ q>q^*\end{cases}. \quad (14)$$

So service provider's profit function ($\Pi(p^*(q),q)$) is increasing-to- decreasing with q.

Since $q\geq 0$, when $\frac{1}{\lambda}\geq\frac{w}{2c\varepsilon}$, $q^*=\frac{1}{\lambda}\text{In}\frac{2c\varepsilon}{w\lambda}$; when $\frac{1}{\lambda}<\frac{w}{2c\varepsilon}$, $q^*=0$.

Proof of lemma 3

Since supply is less than demand, the service provider's profit function is

$$\Pi_S(q)=S(q)[p-v-C(q)]=\xi e^{\mu q}\left(p-v-\frac{2ce^{-\lambda q}}{\lambda^2}\right). \quad (15)$$

Then, the first derivative of $\Pi_S(q)$ is

$$\frac{d\Pi_S(q)}{dq}=\frac{\xi e^{\mu q}}{\lambda^2}\left[2ce^{-\lambda q}(\lambda-\mu)+\mu\lambda^2(p-v)\right]. \quad (16)$$

(1) Since $p-v>0$, $2ce^{-\lambda q}(\lambda-\mu)+\mu\lambda^2(p-v)>0$. When $\lambda-\mu\geq 0$. Then, $\frac{d\Pi_S(q)}{dq}>0$, which means that the service provider's profit function maximizes at $q_S^*=q^\#$.

(2) When $\lambda-\mu<0$, according to the first order condition of $\Pi_S(q)$, we can obtain that

$$q_s^{'}=\frac{1}{\lambda}In\frac{2c(\mu-\lambda)}{\mu\lambda^2(p-v)}. \quad (17)$$

When $q<q_s^{'}$, owing to $2ce^{-\lambda q}(\lambda-\mu)+\mu\lambda^2(p-v)<0$, $\frac{d\Pi_S(q)}{dq}<0$. When $q\geq q_s^{'}$, owing to $2ce^{-\lambda q}(\lambda-\mu)+\mu\lambda^2(p-v)\geq 0$, $\frac{d\Pi_S(q)}{dq}\geq 0$. So the service provider's profit function is decreasing in $\left[0,q_s^{'}\right)$, and increasing in $\left[q_s^{'},q^\#\right]$. Since

$$\begin{cases}q^\#>0\\ \Pi_S(q=0)=\xi\left(p-v-\frac{2c}{\lambda^2}\right)\\ \Pi_S(q=q^\#)=\xi e^{\mu q^\#}\left(p-v-\frac{2ce^{-\lambda q^\#}}{\lambda^2}\right)\end{cases} \quad (18)$$

and $\Pi_S(q=0)<\Pi_S(q=q^\#)$, the service provider's profit function maximizes at $q_S^*=q^\#$.

Combining (1) and (2), it can be inferred that when the supply is less than the demand, $q\leq q^\#$, there is an optimum in $\left[0,q^\#\right]$, which is $q_S^*=q^\#$, that can maximized the provider's profit function $\Pi_S(q)$. Specifically, If the actual quality defect is low ($\frac{1}{\lambda}\leq\frac{1}{\mu}$), the provider's profit function monotonically increases with promised quality defect. If the actual quality defect is high ($\frac{1}{\lambda}\leq\frac{1}{\mu}$), the provider's profit first decreases and then increases with the increase of promised quality defect.

References

[1] Jens H, Dwayne D G 2009 Twenty years of service guarantee research a synthesis *Journal of Service Research* **11**(4) 322-43

[2] Steven A C, Daniel C Q 2008 Exotic reservations-Low-price guarantees *International Journal of Hospitality Management* **27**(2) 162-69

[3] Jehn Y W, Sheng H T, Chih H W 2009 *The Service Industries Journal* **29**(9) 1261-72

[4] Sara B L, Per S 2003 *International Journal of Service Industry Management* **14**(1) 36-58

[5] Rajiv K 2001 The effects of service guarantees on external and internal markets *Academy of Marketing Science Review* **5**(10) 1-19

[6] Timothy L U 2009 *European Journal of Operational Research* **196**(3) 959-67

[7] Tim B, David A C 2005 *Decision Sciences* **36**(2) 197-220

[8] Wen-Chyuan Chiang, Gangshu Cai, Xiaojing Xu, Xiangfeng Chen 2013 Service guarantee and optimal payout models *International Journal of Production Economics* **141**(2) 519-28 *(In chinese)*

[9] Kut C S, Jing S S 1998 *European Journal of operational research* **111**(1) 28-49

[10] Liu W H, Xie D 2013 *International Journal of Production Research* **51**(5) 1618-34

[11] Gordon H M, Terrence L, Peter V 1998 Designing the service guarantee: unconditional or specific *Journal of Services Marketing* **12**(4) 278-93

[12] Christopher W H, Leonard A S, Dan M 1991 Guarantees come to professional service firms *Sloan Management Review* **33**(3) 19-20

[13] Julie M H, Arthur V H 2006 *Journal of Operations Management* **24**(6) 753-64

[14] Vernon N H, Susan H X, Boris J 2009 *Manufacturing & Service Operations Management* **11**(3) 375-96

[15] Jochen W, Doreen K, Khai S L 2000 *Journal of Services Marketing* **14**(6) 502-12

[16] Julie M H, Arthur V H 2001 *Production and Operations Management* **10**(4) 405-23

[17] Hemant K B, Daewon S 2008 *European Journal of Operational Research* **191**(3) 1189-204

[18] Cedric H W, Hsiao C L, Kuang P H, Yi H H 2012 *International Journal of Hospitality Management* **31**(3) 757-63

[19] Howard M, Dan S, Walfried M L 2001 *Journal of Services Marketing* **15**(2) 147-59

[20] Arthur V H, Julie M H, Eitan N 2000 *Journal of Service Research* **2**(3) 254-64

[21] Michael W K, Woody M L 1994 Estimating hidden quality costs with quality loss functions *Accounting Horizons* **8**(1) 8-18

[22] Andrea S, Vince T 2006 *International Journal of Quality & Reliability Management* **23**(6) 647-69

[23] Li M C 2003 Quality loss functions for the measurement of service quality *The International Journal of Advanced Manufacturing Technology* **21**(1) 29-37 *(In chinese)*

[24] Chang H L, Byong D R, Cheng T C 2013 *European Journal of Operational Research* **228**(3) 582-91

[25] Stephen P B, Lieven V 2004 Convex optimization *Cambridge university press*

Author

Wang Wenlong, born in September, 1986, Baoji County, Shaanxi Province, P.R. China

Current position, grades: the Doctoral Student of School of Management, Xi'an Jiaotong University, China.
University studies: received his B.Sc. in Electric Business from Northwest A&F University in China. He received his M.Sc. from Xi'an Jiaotong University in China.
Scientific interest: His research interest fields include service operation, service innovation.
Experience: He participated in many research projects about Service Management from 2010 to 2013, mainly responsible for modelling work.

Liu Xinmei, born in July, 1962, Langfang County,Hebei Province, P.R. China

Current position, grades: the Professor of School of Management, Xi'an Jiaotong University, China.
University studies: received her B.Sc. in Metallograph from Xi'an Jiaotong University in China. She received her M.Sc. and PHD. from Xi'an Jiaotong University in China.
Scientific interest: Her research interest fields include service management and innovation.
Publications: more than 50 papers published in various journals.
Experience: She has teaching experience of over 30 years, has completed several scientific research projects.

Zhang Xiaojie, born in November, 1989, Weifang County, Shandong Province, P.R. China

Current position, grades: the Doctoral Student of School of Management, Xi'an Jiaotong University, China.
University studies: received her B.Sc. in Management from Lanzhou University in China. She received her M.Sc. from Xi'an Jiaotong University in China.
Scientific interest: Her research interest fields include service innovation and creativity.
Experience: She participated in many research projects about service innovation and creativity.