Study on the stability of hybrid systems

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Abstract

Hybrid dynamic system HDS (Hybrid Dynamic Systems) is also called hybrid system, which is composed of event driven subsystem and the time evolution of the interaction subsystem and a class of complex dynamic system. "Mixed" means the combination of continuous and discrete part of the hybrid dynamic system, means that the system dynamic behavior is decided by the interaction of continuous dynamics and discrete dynamics. So far, the hybrid system definition is quite acceptable for: contains discrete event dynamic systems DEDS (Discrete Event Dynamic Systems) and continuous variable dynamic systems CVDS (Continuous Variable Dynamic Systems), system and interaction between them is called the hybrid dynamic system.

Keywords: impulsive switched systems, hybrid system, stability

1 Introduction

The research object of the hybrid system is quite extensive, for example, in the manufacturing industry, according to the machining process, the parts will be transferred to a specified processing machine, process machine to complete this part machining is a dynamic process of time driven, but only in parts arrived, the machine began processing operation, it belongs to event driven, so the entire manufacturing process includes both time driven and event driven dynamics including dynamic, so the whole optimization is considered in the manufacturing process, it needs the dynamic behavior using hybrid system to a more complete detailed portrait of the whole manufacturing system. The hybrid system is also used for interactive programming algorithm and continuous process of discrete, which provides analysis of the basic framework and method for the intelligent system are integrated. In fact, for intelligent design has high ability of self control system, the focus of research in hybrid systems is designed for the continuous supervision system controller.

In addition, the hybrid system in a hierarchical structure to describe complex systems is also useful when, for example, chemical process control systems, air traffic management system and computer communication networks. In some cases, the controlled object is itself a Hybrid system, or is the use of the hybrid controller, which makes the whole system into equivalence of hybrid system. Sometimes in order to simplify the problem, some system is considered as a hybrid system, for example, in order to avoid the direct treatment of complex nonlinear equations, tend to replace it with a simple set of equations (such as linear equations), then switch between these simple equation, which is a commonly used method of physical modeling, a typical example is switching system. Switching system is widely used, such as variable structure system of VSS (Variable Structure System) belongs to the switching system, becomes a big problem in control field is still structure sliding mode control, and is used widely in engineering on the partition of PID, and the Fuzzy control, can be classified as switching system. Hybrid system nearly ten years the attention control and computer theory, because no matter from the point of view of application or theoretical research has the important value. First of all, the hybrid system is widely used in the actual production, such as variable structure control system, mode switching system, a computer control system, batch chemical processes, robotic system, intelligent highway system (Intelligent Vehicle/Highway Systems), modern flight control system (Modern Flight Control Systems) and all at the same time involved logic decision and system of continuous control. In the aspect of theory research, the continuous time dynamic systems usually adopt the differential and difference equations for modeling and analysis, aimed at capturing system "physical" behavior; and for the discrete event dynamic system, usually by the finite automaton representation, aims to capture the system logic or sequential behavior. Because of the two kinds of dynamic characteristics based on different description of mathematical basis, to establish an ideal model of hybrid system contains such two kinds of dynamic characteristics of interaction, mutual influence, analysis of the expected performance, and control the implementation of appropriate, in theory is facing great challenges. Based on the above reasons, the international academic research on the hybrid dynamic system pay more attention to.

The characteristics of hybrid systems can be summarized as follows:

1) system of memory in different properties of continuous and discrete two variables;

2) time and events common to drive the states of the system evolution;

3) continuous variables across the threshold that the state is enabled or disabled;

4) to change the rate of change of continuous variables follow the changes of the discrete state;

5) discrete events in discrete time steps, with sequence, choice, concurrency features;

6) state was the stage, intermittent changes, dynamic characteristics significantly;

7) on the control performance of the system for the

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integrated control of continuous and discrete states;

8) to optimize the performance of system in the integrated optimization of qualitative/quantitative double index.

In short, most of the existing control system can be viewed as a hybrid system, hybrid system theory has been applied in robot, automobile engine, tank level control, hard disk drive, embedded system, network control system, industrial manufacturing, and other places, and has received more and more attention, become a hot research together, automatic control, computer science applied mathematics and other fields.

2 Stability analysis

Analysis and control of hybrid dynamic systems is a cross of Applied Mathematics, system science, control science and computer science, disciplinary study field. In recent years with the development of information industry, especially the rapid development of high-speed computers, international research on complex hybrid system has become increasingly active. A complex hybrid system can also contain some kind of continuous process and some discrete event process, the nature of these various processes occur alternately, interaction. In the complex system of management and control, communication network, artificial intelligence, digital control system, automatic driving design, production line and computer aided control, commercial, industrial and military fields there are a lot of complex hybrid system. Especially those on the system safety requirements high, such as aircraft, automobiles, rockets, missiles, nuclear power plant, the design of the extremely detailed oversight will have catastrophic consequences or the huge economic loss, but such systems are often complex hybrid system.

Switching system is a typical hybrid system. Generally speaking, it consists of a series of subsystems and certain switching rules, among subsystem may be stable, also may be unstable; switching rules may be fixed, but also may be random. So the relevant switching system stability problem is very complicated. Switched systems are of such a nature: even if each subsystem is unstable, we construct some special switching rules, can ensure that the whole system is stable; on the contrary, even if each subsystem is stable, if not suitable choice of switching rules, system may be unstable.

3 Pulse width modulation (PWM) converter analysis

3.1 PWM CONVERTER

PWM converter is the DC voltage can be changed into approximate sine wave voltage high frequency rectangular wave or, after rectifying and filtering step into another DC voltage. It is widely applied to the electronic computer and other electronic equipment UPS (uninterruptible power supply), as large as a distributed power system for ships, aircraft, such as the small portable computer power supply etc.

3.2 THE SYSTEM MODEL DESCRIPTION

A class of PWM switching converter common simplified circuit diagram as shown in Figure 1. Circuit is composed of two parallel loop circuit, including inductance $L_1(r_{L_1})$, $L_2(r_{L_2})$, Switch S_1, S_2, Diode D_1, D_2, resistance and capacitance $C(\mathrm{r}_c)$. V_{in} is the input voltage, it is not a variable.

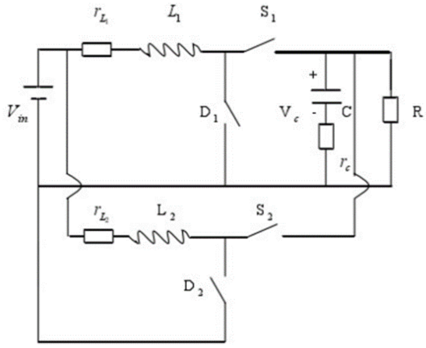

FIGURE 1 Circuit diagram of PWM switching converter

When the two loop circuit operating in continuous conduction state, the diode cutoff, equivalent to D_1, D_2 switch off. That is to say when the switch is turned on, the corresponding diode S_i, D_i end, $i=1$, 2. Therefore, the system has a total of four possible switching mode. Use i_1, i_2 respectively through inductor L_1, L_1, use V_C express capacitor C both ends of the voltage. Assume that the system state variables are $x=[V_C \quad i_1 \quad i_2]^T$, we get the following switching system:

1) S_1, S_2 all off:

$$x=\begin{bmatrix} -\frac{1}{C(R+r_c)} & 0 & 0 \\ 0 & -\frac{r_{L_1}}{L_1} & 0 \\ 0 & 0 & -\frac{r_{L_2}}{L_2} \end{bmatrix} x+\begin{bmatrix} 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} V_{in}.$$

2) S_1 close, S_2 off:

$$x=\begin{bmatrix} -\frac{1}{C(R+r_c)} & 0 & \frac{R}{C(R+r_c)} \\ 0 & -\frac{r_{L_1}}{L_1} & 0 \\ -\frac{R}{L_{2(R+r_c)}} & 0 & -\frac{1}{L_2}\left(\frac{r_c R}{R+r_c}+r_{L_2}\right) \end{bmatrix} x+\begin{bmatrix} 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} V_{in}.$$

3) S_1 off, S_2 close:

$$x = \begin{bmatrix} -\frac{1}{C(R+r_c)} & \frac{R}{C(R+r_c)} & 0 \\ -\frac{R}{L_{2(R+r_c)}} & -\frac{1}{L_2}\left(\frac{r_c R}{R+r_c}+r_{L_2}\right) & 0 \\ 0 & 0 & -\frac{r_{L_1}}{L_1} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} V_{in}.$$

4) S_1, S_2 all close:

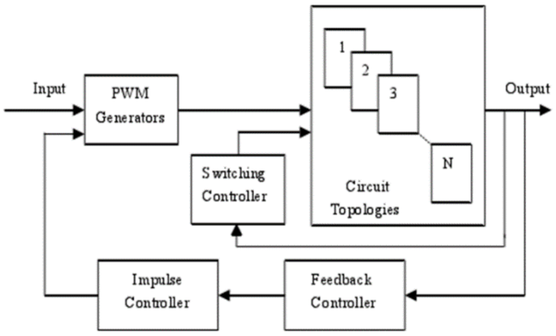

FIGURE 2 The topological graph of Impulsive control of closed loop control system

$$x = \begin{bmatrix} -\frac{1}{C(R+r_c)} & \frac{R}{C(R+r_c)} & 0 \\ -\frac{R}{L_1(R+r_c)} & -\frac{1}{L_1}\left(\frac{r_c R}{R+r_c}+r_{L_1}\right) & 0 \\ -\frac{R}{L_1(R+r_c)} & 0 & -\frac{1}{L_2}\left(\frac{r_c R}{R+r_c}+r_{L_2}\right) \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} V_{in}.$$

Through the above analysis, we can abstract structure diagram of PWM switching converter is as follows:

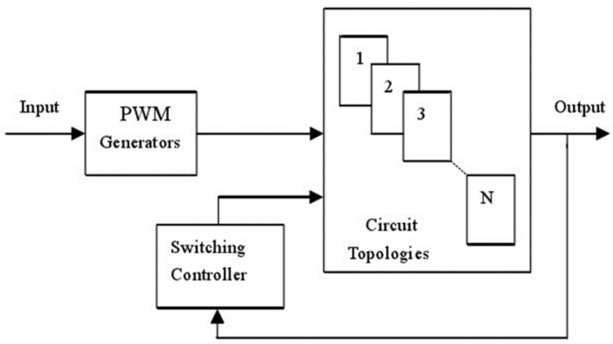

FIGURE 3 Schematic diagram of PWM switching converter

Essentially, PWM switching power converter is composed of different sub circuit topological structure of a series of, these sub circuit is controlled by switching the switch. In order to facilitate the analysis. In order to facilitate the analysis later, we summed up the mathematical expression of this class of hybrid systems are as follows:

$$i \in P\{1,2,...,N\}, \tag{1}$$

where the state variables are a series of appropriate dimension matrix. Based on the PWM switching converter, we add the pulse control and feedback control, get the principle diagram of the controlled system.

Assuming the feedback control system for the evolution of mathematical expressions that:

$$\begin{cases} \dot{x}(t) = (A_i + BiKi)x(t), t \neq t_k \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = E_k x(t_k) \end{cases}, \tag{2}$$

where $i \in \underline{P}$, $x(t): R \to R^n$. As the state vector $k = 1, 2, \ldots$,

$x(t_k) = x(t_k^-) = \lim_{h\to 0} x(t_k - h)$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k - h)$. t_k is K switching time of the symsterm, $0 < t_1 < \ldots < t_k < \ldots$, when B_i is a series of appropriate dimension matrix, is a pulse control matrix.

4 The stability analysis of PWM converter

4.1 BASICAL KNOWLEDGE

Definition. Equation (2) is a stable system, if there exists a switching rule to make the system at the equilibrium $X = 0$ is asymptotically stable:

1) $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} < 0$,

2) $C - B^T A^{-1} B < 0$,

3) $A - BC^{-1}B^T < 0$.

where A, B, C is the appropriate dimension constant matrix.

Considering the controlled PWM converter system Equation (2), P_i is the matrix of appropriate dimensions, define:

$$\Omega_i = \left\{ x \in R^n \middle| x^T \left(P_i A_i + A_i^T P_i + K_i^T B_i^T P_i + P_i B_i K_i\right) x < 0 \right\}. \tag{3}$$

For the Equation (2), if there is a series of matrices ΔE_k and positive definite matrix:

$$\bigcup_{i=1}^{N} \Omega_i = R^n \setminus \{0\}, \tag{4}$$

and for any $i, j \in P$, $k = 1, 2, ...$, the following inequality always is meet:

$$\begin{bmatrix} -P_i & (I+E_k)^T P_j \\ P_j(I+E_k) & -P_j \end{bmatrix} < 0. \tag{5}$$

Then the system is stable:

Hypothesis $\overline{\Omega}_1 = \Omega_1$, $\overline{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i=1} \overline{\Omega}_j$, where $i = 2, 3, \ldots, N$.

Then: $\bigcup_{i=1}^{N} \overline{\Omega}_i = \bigcup_{i=1}^{N} \Omega_i = R^n \setminus \{0\}$, $\overline{\Omega}_j = \Phi$, $i \neq j$.

The operation of the system in the following switching rules: for any $i \in \underline{P}$, When the system state variables $x(t)$ Into the area $\overline{\Omega}_i$, Equation (2) switch to the subsystem i.

Hypothesis $\Psi_t =$ {Subsystem i activation time}, $\varepsilon_i(t) = \begin{cases} 1 & t \in \psi_{t_i} \\ 0 & t \notin \psi_{t_i} \end{cases}$. We construct Lyapunov Function $V(x(t)) = \sum_{i \in p} \varepsilon_i(t) x(t)^T P_i x(t)$, where $i \in \underline{P}$.

For arbitrary switching interval $(t_k \quad t_{k+1})$, Hypothesis $x(t) \in \overline{\Omega}_i$, then $V(x(t)) = x(t)^T P_i x(t)$

so

$$V(x(t)) = x(t)^T P_a \dot{x}(t) + \dot{x}(t)^T P_a x(t) =$$
$$x(t)^T \left(P_i A_i + A_i^T P_i + K_i^T B_i^T P_i + P_i B_i K_i\right) x(t) < 0, \quad (6)$$
$$(x(t) \neq 0)$$

Under arbitrary switching point k, assume that the Equation (2) from the subsystem i switching access subsystem j, then we have

$$V(x(t^+)) + V(x(t^+)) = x(t_k^+)^T P_j x(t_k^+) + \dot{x}(t_k^+)^T P_i x(t_k^+) =$$
$$x(t_k)^T (1+E_k)^T P_j (1+E_k) x(t_k) - \dot{x}(t_k)^T P_i x(t_k) =$$
$$x(t_k)^T \left((1+E_k)^T P_j (1+E_k) - P_i\right) x(t_k),$$

According to the conditions of Equation (5), we get: $(1+E_k)^T P_j (1+E_k) - P_i < 0$, so

$$V(x(t^+)) + V(x(t^+)) < 0, \quad x(t) \neq 0. \quad (7)$$

By the Equations (6) and (7), Equation (2) stable.

Further analysis, if all of the positive definite matrix ($i \in P$) is the same, make, obviously, the establishment of conditions Equation (5), then we get the following corollary: for the Equation (2), if there is a series of matrix, and a positive definite matrix P, make:

$$\bigcup_{i=1}^{N} \Omega_i = R^n \setminus \{0\}, \quad (8)$$

then the Equation (2) in the absence of any impulsive control is stable. Theoretically, Equation (8) is correct, but the state trajectory $x(t)$ may be in the $\overline{\Omega}$, edge repeatedly switch, this is we must avoid frequent switching in practical application. In order to avoid the occurrence of such a situation, we propose the following theorem.

Definition: $\overline{\overline{\Omega}} = \left\{x \in R^n \middle| x^T\left(P_i A_i + A_i^T P_i + K_i^T B_i^T P_i + P_i B_i K_i\right) x < -\partial_i x^T x\right\}$, P_i is positive definite matrix, A_i is Normal number and $i \in \underline{P}$.

For the Equation (2), if there is a series of normal number ∂_i, matrix K_i and the positive definite matrix, make:

$$\bigcup_{i=1}^{N} \overline{\overline{\Omega}}_i = R^n \setminus \{0\}, \quad (9)$$

and for any $i, j \in P$, $k = 1, 2, \ldots$, the following equation:

$$\begin{bmatrix} -P_i & (I+E_k)^T P_j \\ P_j (I+E_k) & -P_j \end{bmatrix} < 0. \quad (10)$$

Then the Equation (2) is stable and can effectively avoid the frequent switching.

$\bigcup_{i=1}^{N} \overline{\Omega}_i = \bigcup_{i=1}^{N} \Omega_i = R^n \setminus \{0\}$, $\Omega_1 = \quad \Omega_1 - \bigcup_{j=1}^{i-1} \Omega_j$, $i = 2, 3, \ldots, N$.

Switching rules are as follows: assume that the Equation (2) run in any subsystem i, $i \in P$. When the system state $x(t)$ ran out of area, phylogenetic switch and switch to the subsystem j, here.

Below we mainly discussed above switching rules, Equation (2) is how to avoid the frequent switching occurs.

Assume that the Equation (2) from the system i switch to the subsystem j then $x(t^+) \in \Omega_j$, we have: $\left(P_i A_i + A_i^T P_i + K_i^T B_i^T P_i + P_i B_i K_i\right) x < -\partial_i x^T x$.

Then, $x(t^+) \in \Omega_j$, $x(t^+)$ is far away from the area Ω_j boundary. State trajectory must spend some time to from $x(t^+)$ move to Ω_j 's boundary. So, the switching system will not happen immediately. This can effectively avoid the occurrence of frequent switching.

4.2 EXAMPLE

Consider a two-dimensional controlled systems, and the system is switched in two linear subsystems, that is $i = 2$.

$$\dot{x}(t) = A_i x(t) + B_i u, t \neq t$$
$$\Delta(t_k) = x(t_k^+) - x(t_k^-) = E_k x(t_k)$$
$$A_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad . \quad (11)$$
$$A_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Obviously, for each individual subsystem above, were added to any feedback control $u = K_x$, subsystem is still not stable. We select the following state feedback matrix: $K_1 = [-2 \quad -2], K_2 = [-2 \quad -2]$.

Equation (11) into:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} x(t), t \neq t_k \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = E_k x(t_k) \end{cases}, \quad (12)$$

and

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x(t), t \neq t_k \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = E_k x(t_k) \end{cases}. \quad (13)$$

Make the positive definite matrix, $P_1 = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$, positive , $\partial_2 = 0.5$, we have $\overline{\overline{\Omega}}_2 = \{(x_1, x_2) | -2x_1^2 + x_2^2 < 0\}$ and $\overline{\overline{\Omega}}_1 = \{(x_1, x_2) | x_1^2 - x_2^2 < 0\}$, $\overline{\overline{\Omega}}_2 = \{(x_1, x_2) | -3.5x_1^2 + 2.5x_2^2 < 0\}$.

Obviously $\tilde{\tilde{\Omega}}_1 = \overline{\overline{\Omega}}_1$, $\tilde{\tilde{\Omega}}_2 = \overline{\overline{\Omega}}_2$, then $\tilde{\tilde{\Omega}}_1 \cup \tilde{\tilde{\Omega}}_1 = \overline{\overline{\Omega}} \cup \overline{\overline{\Omega}} = R^2 \setminus \{0\}$, $\tilde{\tilde{\Omega}}_1 \cap \tilde{\tilde{\Omega}}_1 = \Phi$. So the switching rules. Through the solution of Equation (12).

We get the pulse control matrix, $k = 1, 2, \ldots$ so the Equation (13) is stable. Figure 4 shown above the controlled system from the initial value, $x_2(0) = 4$ the beginning of the state response; Figure 5 shows the system's phase.

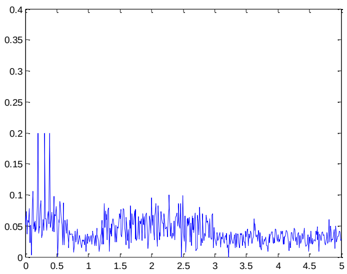

FIGURE 4 The controlled system state response

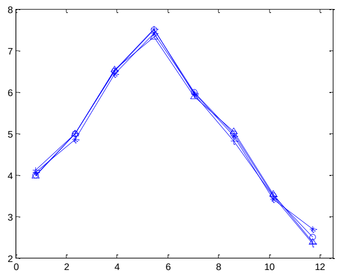

FIGURE 5 The phase diagram of the controlled system

5 Conclusion

In this Article, we take the PWM converter as an example, according to its working principle from the actual circuit extracts the mathematical expression of the abstract, then using the Lyapunov function method, the stability problem to the study of the controlled system controlled by pulse, and obtain some sufficient conditions of stability of some system, finally, a numerical example verifies the effectiveness of the conclusion.

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