

Optimal acquisition and pricing policies for remanufacturing systems with initial investment

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Abstract

The problem of used product (core) acquisition is an important issue in remanufacturing. In traditional models, remanufacturing systems are assumed to be well-established. However, remanufacturing systems in most developing countries are imperfect and remanufacturers need to make huge initial investments to improve the remanufacturing systems. It is therefore suggested that the effects of initial investment cannot be neglected. In this paper, an acquisition and pricing problem in imperfect remanufacturing systems is studied. The problem is firstly formulated as a two-period nonlinear programming model, and the closed forms of the optimal solution are presented based on Karush-Kuhn-Tucker conditions. Next, the multi-period acquisition and pricing problem, and the effects of the initial investment are discussed. Finally, the conclusions are testified by numerical examples. The results show that with the remanufacturing system improved, the remanufacturer will increase investment and acquire more cores.

Keywords: Core acquisition, Imperfect remanufacturing systems, Nonlinear programming model, Karush-Kuhn-Tucker conditions

1 Introduction

Remanufacturing is growing into a booming industry because of its resource sustainability and environmental friendliness. Remanufacturing driven by cost reduction has become an important technological field and claims a large market share, especially in developed countries. However, in developing countries (e.g., Brazil, China, and India), there are different levels of development in remanufacturing. Remanufacturing systems in developing countries suffer from limitations, such as a poor recycling network, deficiency in regulation, low customer recognition and low environmental awareness. These limitations reduce the profitability and enthusiasm of firms. In order to improve remanufacturing systems, the initial investment should be made based on the recovery network, remanufacturing technique, recovery advertising, and so on. The well-established remanufacturing system depends on huge initial investment and long-term development, and consequently most firms undertake their remanufacturing activities on the grounds of environmental responsibilities rather than economic benefits.

Motivated by the above observations, an acquisition and pricing problem is studied in the presence of initial investment. In order to determine the optimal policies, the two-period and multi-period models are developed respectively. Moreover, the effects of the initial investment on the optimal policies are analysed. Finally, numerical experiments are provided to validate the analysis conclusions.

The rest of this paper is organized as follows: In section 2, related research is briefly reviewed. In section 3, the problem is described and a two-period model is formulated. Additionally, the effects on the optimal policies are analysed. In section 4, the model and the conclusions are extended to a multi-period model. In addition, the numerical examples are illustrated in section 5. Finally, section 6 concludes the paper with future research directions.

2 Literature review

There is an abundance of literature related to remanufacturing, which can be categorized into different research themes including inventory management [1, 2], production planning and control (PP&C) [3], capacity planning [4, 5], product acquisition [6, 7], and network design [8, 9].

The literature related to our work is reviewed. Debo et al., Heese et al., and Ferguson and Toktay studied the policies as to whether to manufacture new products and remanufactured products or not [10, 12]. Ferrer and Swaminathan developed two-period, multi-period, and infinite planning models respectively to identify thresholds in remanufacturing operations in the monopoly and duopoly environment [13]. These literature assumed that the new and remanufactured products were undistinguishable. Ferrer and Swaminathan further extended the model to the case in which consumers could differentiate between remanufactured products and new products [14]. Webster and Mitra studied a general two-

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period model to investigate the effects of take-back laws on remanufacturing [15]. Additionally, Mitra and Webster continued their work to focus on the impacts of government subsidies [16].

In the existing literature, it is assumed that the remanufacturing system is well-established with a perfect recovery network and that consumers have a high awareness of recycling. Apparently, these assumptions are in conflict with the real situation, which is that firms have high incentive to remanufacture cores, especially in developing countries. In these developing countries, due to the imperfect remanufacturing system, the impact of initial investment cannot be neglected. In addition, the existing literature assumes that the return rate is exogenous and is generally set to the statutory return rate. However, in the imperfect remanufacturing system, the statutory return rate could not be set or strictly enforced. Firms could determine the quantity of the remanufactured products, and the return rate is no longer exogenous. In this paper, the initial investment is incorporated into a pricing problem for a remanufacturing system. Most importantly, it is focused on the impacts of the initial investment on optimal policies.

3 The two-period acquisition and pricing model

3.1 PROBLEM DESCRIPTIONS AND ASSUMPTIONS

This paper assumes that the analysis is applied in an imperfect remanufacturing system. In this remanufacturing system, a monopolist firm manufactures new products in the first period and makes both new products and remanufactured products in the future period. For the multi-period case, there are M planning horizons. At the beginning of the first period, the firm determines the optimal acquisition and pricing of each period. The analysis is based on the following assumptions:

Assumption 1. There is no distinction between new products and remanufactured products. That is, consumers cannot distinguish remanufactured products from new products, and the firm charges the same prices for both products.

Assumption 2. Q is the size of the potential market, which is constant in each period. c is the unit cost of new products. s is the unit cost saving of remanufactured products, so the unit cost of remanufactured products is $c - s$.

Assumption 3. p_{ij} is the unit price of j type of product in period i ($i = 1, 2, \dots, M$), and q_{ij} is the quantity of the corresponding product type. Subscript j will take the values N, R , and A , denoting new products, remanufactured products and all products, respectively.

Note that the subscript j is omitted in the first period with only new products.

Assumption 4. In period i , remanufactured products are made of cores sold in period $i-1$, and will be remanufactured and sold in the present period.

Let p_i ($i = 2, \dots, M$) denote the core return rate in period i , $0 < \rho_i < 1$. Therefore, $q_{2R} \leq \rho_2 q_1$ in the two-period case and $q_{iR} \leq \rho_i (Q - p_{i-1})$ in the multi-period case.

Since the remanufacturing system is deficient, the firm should invest in order to improve the system. Firms have little incentive to remanufacture products. The return rate is endogenous variable. Furthermore, the less effective the system, the higher the investment for the same return rate. Hence, the return rate function proposed by Savaskan et al. is extended [17].

Assumption 5. The function between the investment and the return rate is $I_i = (K_i / 2) \rho_i^2$ in period i ($i = 2, \dots, M$). K_i is a scaling parameter, which represents the investment per return rate. It is obvious that the scale parameter decreases with the improvement of the remanufacturing system. That is, $K_{i-1} > K_i$.

Assumption 6. The demand function is linearly decreasing in price. Hence, the demand for period i is $q_{iA} = q_{iN} + q_{iR}$. Additionally, β denotes the discount factor per period, and $0 < \beta < 1$.

To avoid trivial cases, it is assumed that $Q > c$, $c > s$, $2K_i > \beta s^2$.

3.2 THE MODEL FORMULATION AND PROPERTIES

In the two-period case, the firm manufactures new products in the first period and manufactures both new and remanufactured products in the second period. The quantity of new products in the first period is given by $q_1 = Q - p_1$, and the total quantity of new products and remanufactured products in the second period is given by $q_{2A} = q_{2N} + q_{2R} = Q - p_2$. Hence, considering the investment in the remanufacturing system, the model can be expressed as:

(Model P1)

$$\max_{p_1, p_2, q_{2N}, q_{2R}} \Pi_{2R} = (p_1 - c)q_1 + \beta \{ (p_2 - c)q_{2N} + (p_2 - c + s)q_{2R} - K \rho^2 / 2 \}, \tag{1}$$

s.t.:

$$q_{2R} \leq \rho_2 q_1 \tag{2}$$

In P1, $(p_1 - c)q_1$ and $(p_2 - c)q_{2N}$ denote the profits on new products in the first period and the second period respectively. $(p_2 - c + s)q_{2R}$ is the profit on remanufactured products in the second period, and the subscript in this term ($i = 2$) is omitted.

Let λ be the according dual variable of constraint (2). The Karush-Kuhn-Tucker (KKT) conditions of the acquisition problem can then be rewritten as follows:

$$(Q - p_1) - (p_1 - c) - \lambda\rho = 0, \tag{3}$$

$$\beta s - \lambda = 0, \tag{4}$$

$$-\beta k\rho + \lambda(Q - p_1) = 0, \tag{5}$$

$$\lambda[\rho(Q - p_1) - q_{2R}] = 0. \tag{6}$$

Thus, the optimal solution is shown in Table 1.

TABLE 1 The optimal solution of P1

Policies	Period 1	Period 2
Pricing	$p_1 = \frac{Q+c-\beta s\rho}{2}$	$p_2 = \frac{Q+c}{2}$ $q_2 = \frac{Q-c}{2}$
Quantity	$q_1 = \frac{Q-c+\beta s\rho}{2}$	$q_{2N} = \frac{(Q-c)(1-\rho)-\beta s\rho^2}{2}$ $q_{2R} = \frac{(Q-c)\rho+\beta s\rho^2}{2}$
Acquisition		$\rho^* = \frac{s(Q-c)}{2K-\beta s^2}$

Due to $q_{2N} > 0$, $(2K - \beta s^2)^2 > 2Ks(Q - c)$. Next, the effects of initial investment on the optimal solution are analysed.

Proposition 1. (a) The optimal return rate is the decreasing function of K . (b) I is the decreasing function of K .

Proof. (a) As shown in the expressions of ρ^* , the first order condition is:

$$\left. \frac{\partial \rho}{\partial K} \right|_{\rho=\rho^*} = \left. \frac{-2s(Q-c)}{(2K-\beta s^2)^2} \right|_{\rho=\rho^*} < 0.$$

So, ρ^* is the decreasing function of K .

(b) Due to $I = (K/2)\rho^2 = \frac{K}{2} \left[\frac{s(Q-c)}{2K-\beta s^2} \right]^2$, it is easily

concluded that the first order condition $\frac{\partial I}{\partial K} = \left[\frac{s(Q-c)}{2K-\beta s^2} \right]^2 \left[\frac{-2K-\beta s^2}{2K-\beta s^2} \right] < 0$. Therefore, I is the decreasing function of K .

Proposition 1 implies that the lower the scaling parameter, the higher the return rate. Similarly, the lower the scaling parameter is, the higher the total investment. The lower scaling parameter means that the remanufacturing system is well-established. That is, the investment for the same return rate is relatively small. In the market-driven remanufacturing system, the firm has higher incentive to acquire and remanufacture cores. Additionally, the well-established remanufacturing system promotes consumers' awareness of remanufacturing. Hence, the firm increases the

investment on remanufacturing activities and network. This conclusion coheres with the real situation. In developed countries, not only the related laws and regulations, but also the recovery network and consumer awareness are well-established, which leads to a higher optimal return rate, for example, some firms in America and Germany can reach a rate of 90%. While the remanufacturing industry in developing countries is still in the difficult beginning period, the real situation, such as the recovery network, firms' enthusiasm, society consumer awareness, and so on, need to be improved. The return rate is therefore small and it is hard to obtain investment.

Proposition 2. (a) p_1 is the increasing function of K . (b) q_1 is the decreasing function of K . (c) The investment has no impact on p_2 and q_2 . (d) q_{2N} is the increasing function of K , and q_{2R} is the decreasing function of K .

Proof. (a) The first order condition of p_1 could be written as follows:

$$\frac{\partial p_1}{\partial K} = \frac{\partial p_1}{\partial \rho} \cdot \frac{\partial \rho}{\partial K} = \frac{-\beta s}{2} \cdot \frac{-2s(Q-c)}{(2K-\beta s^2)^2} = \frac{-\beta(Q-c)s^2}{(2K-\beta s^2)^2}.$$

Since $\beta > 0$ and $Q - c > 0$, $\frac{\partial p_1}{\partial K} > 0$. That is, p_1 is the increasing function of K .

(b) Similarly,

$$\frac{\partial q_1}{\partial K} = \frac{\partial q_1}{\partial \rho} \cdot \frac{\partial \rho}{\partial K} = \frac{\beta s}{2} \cdot \frac{-2s(Q-c)}{(2K-\beta s^2)^2} = -\frac{\beta s^2(Q-c)}{(2K-\beta s^2)^2} < 0.$$

Hence, q_1 is the decreasing function of K .

(c) As shown in the expressions of p_2 and q_2 in table 1, the change is not dependent on K .

(d) According to the first order condition, we have:

$$\frac{\partial q_{2N}}{\partial K} = \frac{\partial q_{2N}}{\partial \rho} \cdot \frac{\partial \rho}{\partial K} = \frac{-(Q-c)-2\beta s\rho}{2} \cdot \frac{-2s(Q-c)}{(2K-\beta s^2)^2} > 0,$$

$$\frac{\partial q_{2R}}{\partial K} = \frac{\partial q_{2R}}{\partial \rho} \cdot \frac{\partial \rho}{\partial K} = \frac{Q-c+2\beta s\rho}{2} \cdot \frac{-2s(Q-c)}{(2K-\beta s^2)^2} < 0.$$

So, q_{2N} is the decreasing function of K , and q_{2R} is the increasing function of K .

When the scaling parameter is relatively large, the return rate is small. The small return rate implies that the quantity of remanufactured products in the second period is small. The firm therefore has to increase the price in the first period to maximize profits, which consequently leads to production decreasing. Moreover, since the total quantity is not affected by the investment, the amount of new products increases with the remanufactured products decreasing.

4 The multi-period acquisition and pricing model

In fact, due to the huge initial investment, the firm will invest in remanufacturing systems in stages. Hence, this section assumes that the firm has a planning horizon of M periods ($M > 2$). The model for the multi-period planning horizon is therefore given as:
(Model P2)

$$\max_{p_i, q_{iR}, \rho} \Pi = (p_1 - c)q_1 + \sum_{i=2}^M \beta^{i-1} [(p_i - c)q_{iN} + (p_i - c + s)q_{iR} - K_i \rho_i^2 / 2] \quad (7)$$

s.t.:

$$q_{iR} \leq \rho_i(Q - p_{i-1}), \quad i = 2, \dots, M, \quad (8)$$

$$K_{i-1} > K_i, \quad i = 3, \dots, M. \quad (9)$$

The definitions of the terms in P2 are similar to P1. Similarly, based on the KKT conditions, the optimal solution of P2 can be obtained in table 2.

TABLE 2 The optimal solution of P2

Policies	Period 1	Period i ($i=2, \dots, M-1$)	Period M
Pricing	$p_1 = \frac{Q+c-\beta s \rho_2}{2}$	$p_i = \frac{Q+c-\beta s \rho_{i+1}}{2}$	$p_M = \frac{Q+c}{2}$
Quantity	$q_1 = \frac{Q-c+\beta s \rho_2}{2}$	$q_{iN} = \frac{(Q-c)(1-\rho_i)+\beta s(\rho_{i+1}-\rho_i^2)}{2}$ $q_{iR} = \frac{\rho_i(Q-c+\beta s \rho_i)}{2}$	$q_{MN} = \frac{(Q-c)(1-\rho_M)-\beta s \rho_M^2}{2}$ $q_{MR} = \frac{\rho_M(Q-c+\beta s \rho_M)}{2}$
Acquisition			$\rho_i^* = \frac{s(Q-c)}{2K_i - \beta s^2}$

According to the expression of ρ_i^* , Proposition 1 is also valid in the multi-period case. With the improvement of the remanufacturing system, $K_{i-1} > K_i$. According to the principal of profit maximization, the cost savings are attractive to the firm. The firm has incentive to acquire and remanufacture more cores.

Hence, the investment increases with the passing of time ($I_{i-1} < I_i$). The optimal return rate also increases.

Proposition 3. (a) p_i ($i = 1, 2, \dots, M - 1$) is increasing with K_{i+1} , and the investment has no impact on p_M . (b) q_{iR} ($i = 2, \dots, M$) is decreasing with K_i . (c) q_{iN} ($i = 1, 2, \dots, M - 1$) is increasing with K_i , but decreasing with K_{i+1} . In addition, q_{MN} is increasing with K_M .

Proof: In period i ($i = 1, 2, \dots, M - 1$), the first order condition of p_i is:

$$\frac{\partial p_i}{\partial K_{i+1}} = \frac{\partial p_i}{\partial \rho_{i+1}} \cdot \frac{\partial \rho_i}{\partial K_{i+1}} = -\frac{s\beta}{2} \cdot \frac{-2s(Q-c)}{(2K_{i+1} - \beta s^2)^2} > 0, \text{ so } p_i \text{ is the increasing function of } K_{i+1}.$$

Similarly, the other conclusions can be obtained:
In period i ($i = 2, \dots, M$),

$$\frac{\partial q_{iR}}{\partial K_i} = \frac{\partial q_{iR}}{\partial \rho_i} \cdot \frac{\partial \rho_i}{\partial K_i} = \frac{Q-c+2\beta s \rho_i}{2} \cdot \frac{-2s(Q-c)}{(2K_i - \beta s^2)^2} < 0.$$

In period i ($i = 1, 2, \dots, M - 1$),

$$\frac{\partial q_{iN}}{\partial K_i} = \frac{\partial q_{iN}}{\partial \rho_i} \cdot \frac{\partial \rho_i}{\partial K_i} = -\frac{Q-c+2\beta s \rho_i}{2} \cdot \frac{-2s(Q-c)}{(2K_i - \beta s^2)^2} > 0,$$

$$\text{and } \frac{\partial q_{iN}}{\partial K_{i+1}} = \frac{\partial q_{iN}}{\partial \rho_{i+1}} \cdot \frac{\partial \rho_{i+1}}{\partial K_{i+1}} = \frac{\beta s}{2} \cdot \frac{-2s(Q-c)}{(2K_{i+1} - \beta s^2)^2} < 0.$$

Additionally, in period M ,

$$\frac{\partial q_{MN}}{\partial K_M} = \frac{\partial q_{MN}}{\partial \rho_M} \cdot \frac{\partial \rho_M}{\partial K_M} = -\frac{Q-c+2\beta s \rho_M}{2} \cdot \frac{-2s(Q-c)}{(2K_M - \beta s^2)^2} > 0$$

Due to $K_{i-1} > K_i$, it is easily concluded that the price decreases as more time periods pass. Therefore, the demand for the product increases. Since consumers cannot distinguish between new products and remanufactured products, and remanufactured products have larger marginal profit, the increasing remanufactured products will cannibalize the new products market. As time going on, the quantity of new products will decrease while the quantity of remanufactured products will increase.

5 Numerical examples

In this section, numerical examples are presented to validate the analysis conclusions and obtain some managerial insights. Firstly, in the two-period case, the parameters are set as follows: $Q = 10, c = 3.5, \beta = 0.95$,

$s = 1$, and $K \in [5, 23]$. The optimal policies under the different scaling parameters are showed in figure1-4. It is

obvious that Propositions 1 and 2 are justified by Figure1-4.

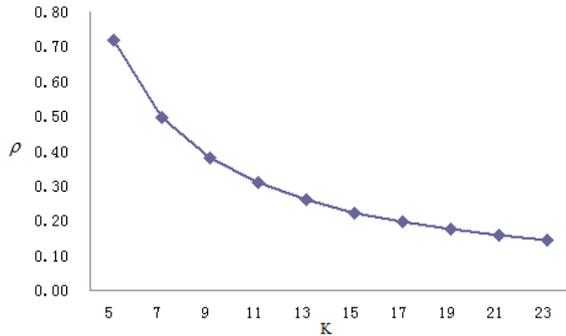


FIGURE 1 The optimal return rate

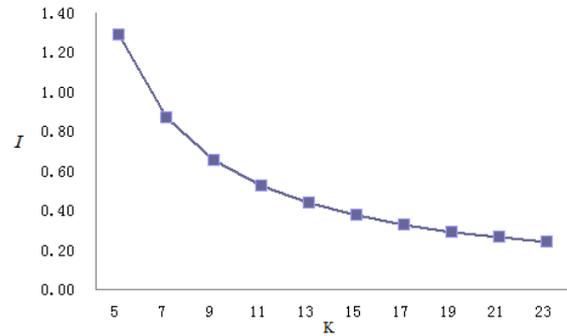


FIGURE 2 The initial investment

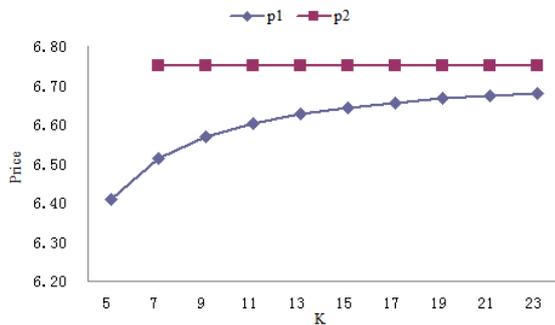


FIGURE 3 The optimal price

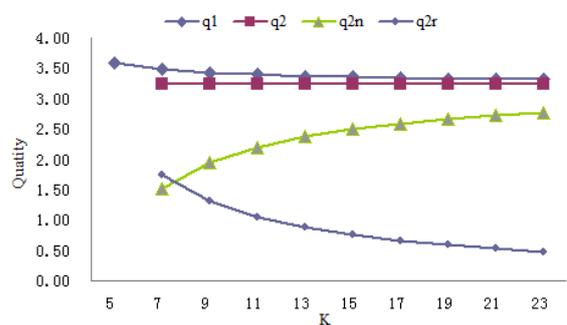


FIGURE 4 The optimal quantities

Next, the eleven-period case is considered. The parameters are set as follows: $Q = 10$, $c = 3.5$, $\beta = 0.95$, $s = 1$, and $K_i \in [4, 40]$. The result is shown in table 3. Table 3 shows that as the scaling parameter decreases from period 1 to 11, the respective optimal return rate

increases. Moreover, the quantity of remanufactured products and the investment in the remanufacturing system increases. With the exception of the last period, the price and quantity of new products decreases. These results cohere with the analytical conclusions.

TABLE 3 The optimal solution in the eleven-period case

Period	K_i	ρ_i^*	p_i	q_n	q_r	I_i	Π_i
1	-	-	7.4716	5.8608	-	-	14.4852
2	40	6.32%	7.4684	2.3718	0.1598	0.1598	20.7342
3	36	7.03%	7.4643	2.3576	0.1780	0.1780	26.9829
4	32	7.92%	7.4592	2.3399	0.2009	0.2009	33.2312
5	28	9.07%	7.4522	2.3172	0.2306	0.2306	39.4790
6	24	10.62%	7.4425	2.2871	0.2705	0.2705	45.7257
7	20	12.79%	7.4277	2.2453	0.3271	0.3271	51.9704
8	16	16.08%	7.4026	2.1838	0.4136	0.4136	58.2109
9	12	21.65%	7.3510	2.0868	0.5622	0.5622	64.4387
10	8	33.11%	7.1831	1.9397	0.8772	0.8772	70.5883
11	4	70.42%	7.5000	0.5163	1.9837	1.9837	76.8383

6 Conclusions

It is essential to consider the investment in remanufacturing systems when assessing the real situation of remanufacturing systems in developing countries. This paper formulated the two-period and multi-period model. Moreover, the relationship between the optimal policies and the scaling parameter was analysed. In addition, the numerical examples illustrated the analytical conclusions. In summary, this paper makes a contribution to the literature on remanufacturing by drawing attention to the real situation in developing countries. For future work, it is natural to consider the

optimal policies in a competitive market. The impact of government subsidy on remanufacturing also deserves attention, e.g., considering the effect of the constraint of lowest return rate regulated by the government.

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