

Data analysis of basketball game performance based on bivariate poisson regression model

Ke Shen*

Physical Education Institute, Hunan University of Technology, Zhuzhou 412007, Hunan, China

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Abstract

Conventional methods used to process two-dimensional discrete data will produce large errors and have a narrow scope of application. Along with the development of mathematical theories and computer technologies, some scholars propose to process two-dimensional discrete data by bivariate Poisson regression model, which takes correlation among data sets into consideration and has excessive variability so that results of data analysis can be more accurate. This paper firstly introduces bivariate Poisson distribution and bivariate Poisson regression model, and then uses this model to analyze performance data of each team in regular seasons of 2013-2014 CBA (China Basketball Association) and 2012-2013 NBA (National Basketball Association), and predict performance in post seasons. Through comparison between actual results and results of double independent Poisson distribution, this model can better predict game performance.

Keywords: Two-dimensional discrete data, Bivariate Poisson distribution, Bivariate Poisson regression model, Basketball

1 Introduction

Two-dimensional data is mainly divided into two categories, namely two-dimensional continuous data and two-dimensional discrete data [1]. The former is mainly processed by bivariate continuous distribution. With the development of mathematical theories and computer technologies, people have basically mastered processing methods of two-dimensional continuous data [2]. However, because of calculation complexity, no progress has been made in terms of processing two-dimensional discrete data [3,4]. At present, there are mainly two methods for processing two-dimensional discrete data: one is approximation of data by bivariate continuous distribution. For example, assume that data obeys bivariate normal distribution, and then bivariate normal distribution processing means can be used to analyze data, so that calculation could be simple [5]. The other is to assume that two data sets in two-dimensional discrete data are mutually independent [6]. In this way, two-dimensional discrete data can be converted into two sets of one-dimensional and mutually independent discrete data, and thus the calculation is simplified. For instance, assume that data set M_1 and M_2 in two-dimensional discrete data sets are mutually independent, and:

$$M_i \sim P\lambda_i, i = 1, 2. \quad (1)$$

After using hypothesis test, two-dimensional discrete data sets can be analyzed according to two independent Poisson distributions, and this method is called double independent Poisson distribution [7].

In basketball games, scores of two teams can be seen as two discrete data sets, and currently basketball scores are

mainly analyzed by the above two processing methods. However, these methods have defects: firstly, process scores of discrete basketball games by approximating as continuous distribution, and then it is often impossible to find corresponding continuous distribution, or large errors may be resulted in approximation because of too little amount of data [8]. Secondly, consider that scores of two basketball teams are mutually independent, so as to analyze performance of each team respectively [9]. But in fact, during games, scoring ability, pace and home-away environment of one side will have an influence on the other. Thus, such practice is not rigorous and is very prone to errors [10].

To solve the above defects, Karlis et al proposed to analyze sports game data by bivariate Poisson regression model in 2003. Bivariate Poisson regression model can process two-dimensional discrete data effectively, and bivariate Poisson distribution that it uses was put forward by Holgate in 1964 [11]. During the development, scholars put forward various methods to generate bivariate Poisson distribution probability density function, including bivariate binomial distribution limit acquisition method and trivariate reduction method etc. Compared with conventional processing means, bivariate Poisson regression model takes correlation among data sets into consideration and has excessive variability, so that results of data analysis can be more accurate and have a wider scope of application [12]. Based on the above advantages, bivariate Poisson regression model has been widely used in planning prior insurance rate, prediction of fertility rate and unemployment rate etc. This paper firstly introduces bivariate Poisson distribution and regression model, and then analyzes basketball game performance data by bivariate Poisson regression model.

* *Corresponding author's* e-mail: shenke111@yeah.net

2 Bivariate Poisson distribution and regression model

Trivariate reduction method was proposed by Kocherlakota in 1992 to calculate probability density function of bivariate Poisson distribution. As this method is widely used, this paper mainly uses trivariate reduction method to analyze and introduce bivariate Poisson distribution and regression model.

Assume that, X_1, X_2, X_3 are random variables that obey Poisson distribution, and are mutually independent with coefficients of λ_1, λ_2 and λ_3 respectively. Build another set of random variables Y_1 and Y_2 , where $Y_1 = X_1 + X_3$ and $Y_2 = X_2 + X_3$, and then joint distribution (Y_1, Y_2) obeys bivariate Poisson distribution. Its probability density function is:

$$\begin{aligned}
 P(Y_1 = y_1, Y_2 = y_2) &= \\
 &= P(X_1 + X_3 = y_1, X_2 + X_3 = y_2) = \\
 &= \exp\left\{-\left(\lambda_1 + \lambda_2 + \lambda_3\right)\right\} * \frac{\lambda_1^{y_1}}{y_1!} * \frac{\lambda_2^{y_2}}{y_2!} * \\
 &* \sum_{i=1}^{\min(y_1, y_2)} \binom{y_1}{i} \binom{y_2}{i} i! \left(\frac{\lambda_3}{\lambda_1 \lambda_2}\right)^i
 \end{aligned} \tag{2}$$

In addition, covariance is $Cov(Y_1, Y_2) = \lambda_3$, expected marginal distribution is $EY_1 = \lambda_1 + \lambda_3, EY_2 = \lambda_2 + \lambda_3$, and marginal variance is $Var(Y_1) = \lambda_1 + \lambda_3, Var(Y_2) = \lambda_2 + \lambda_3$. It can be seen that covariance $Cov(Y_1, Y_2) = \lambda_3$ represents the correlation between two sets of random variables. If $\lambda_3 = 0$ two sets of random variables are mutually independent, and then two-dimensional discrete data set obeys the abovementioned double independent Poisson distribution.

Then, define variable difference as $Z_1 = Y_1 - Y_2$, and probability density function of Z_1 is:

$$P(Z_1 = z) = \exp\left\{-\left(\lambda_1 + \lambda_2\right)\right\} \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{z}{2}} I_z\left(2\sqrt{\lambda_2 \lambda_3}\right) \tag{3}$$

Where, function I is modified Bessel Function. By where, function I is modified Bessel Function. By subtracting probability density function expression of Z_1 and probability density functions of two independent Poisson distribution, the same results can be obtained. However, according to the above analysis, each set of random variables in bivariate Poisson distribution is related to λ_3 , which is not 0, and thus probability of variable difference in bivariate Poisson distribution is actually different from variable difference probability of two independent Poisson distributions.

Define the sum of variables as $Z_2 = Y_1 + Y_2$, and then the expected marginal distribution is $EZ_2 = \lambda_1 + \lambda_2 + 2\lambda_3$, and marginal variance is $Var(Z_2) = \lambda_1 + \lambda_2 + 4\lambda_3$. As λ_3 is

positive, $Var(Z_2) > EZ_2$, and this property is called excessive variability.

Building method of bivariate Poisson regression model is similar to that of single-variable Poisson regression model, namely analyze various impact factors of data, introduce to model as covariates and then analyze actual situation. Take (Y_{1m}, Y_{2m}) as the m^{th} sample data set, where $m=1,2,3,\dots,n$ and each data set obeys bivariate Poisson distribution, introduce parameters $\lambda_{1m}, \lambda_{2m}$ and λ_{3m} to analysis process as covariates, and bivariate Poisson regression model can be obtained as follows:

$$\left\{ \begin{aligned}
 (Y_{1m}, Y_{2m}) &\sim P(\lambda_{1m}, \lambda_{2m}, \lambda_{3m}) \\
 \log(\lambda_{1m}) &= a_m \beta_1 \\
 \log(\lambda_{2m}) &= b_m \beta_2 \\
 \log(\lambda_{3m}) &= c_m \beta_3
 \end{aligned} \right. \tag{4}$$

Where β is regression coefficient vector, a_m, b_m and c_m are covariate vectors with $m=1,2,3,\dots,n$. Values of covariate vectors change according to specific problems, and covariate vectors of parameters $\lambda_{1m}, \lambda_{2m}$ can λ_{3m} be the same or not. If covariate vector values are the same, it is called consistency hypothesis. As observed values and covariate vectors of samples are known, bivariate Poisson regression model can be obtained by merely re-estimating to obtain value of regression coefficient vector. Here, maximum likelihood estimation is mainly used to estimate regression coefficient, and generally no covariate is introduced to parameter λ_3 in order to simplify calculation. Then, logarithm of likelihood function can be expressed as:

$$\log L = -\sum_{m=1}^n (\lambda_{1m} + \lambda_{2m} + \lambda_3) + \sum_{m=1}^n \log \varnothing \tag{5}$$

$$\varnothing = \sum_{j=0}^{\min(y_{1m}, y_{2m})} \frac{\lambda_{1m}^{y_{1m}-j} \lambda_{2m}^{y_{2m}-j} \lambda_3^j}{(y_{1m}-j)!(y_{2m}-j)!j!} \tag{6}$$

So, the obtained maximum value of $\log L$ is the maximum value of likelihood function, and generally the derivative of function shall be 0 in order to get its maximum value. Therefore, take partial derivatives of parameters β_1, β_2 and λ_3 b $\log L$ respectively, and:

$$\left\{ \begin{aligned}
 \frac{\partial \log L}{\partial \beta_1} &= \sum_{m=1}^n \lambda_{1m} u_m (P_{10} - 1) = 0 \\
 \frac{\partial \log L}{\partial \beta_2} &= \sum_{m=1}^n \lambda_{2m} v_m (P_{01} - 1) = 0 \\
 \frac{\partial \log L}{\partial \lambda_3} &= -n + \sum_{m=1}^n P_{11} = 0 \\
 P_{jk} &= \frac{\Phi(y_{1m}-j, y_{2m}-k)}{\Phi(y_{1m}, y_{2m})}
 \end{aligned} \right. \tag{7}$$

3 Application of bivariate Poisson regression model in basketball game data analysis

Cummins et al proposed in 1983 that bivariate Poisson regression model can be applied to insurance claims and clarification of insurance premium etc, which was the earliest application of this model. Compared with conventional double independent Poisson distribution, results obtained through this model are more consistent with actual situation, and its application also obtains good benefits. With the promotion of this model, Karlis et al proposed in 2003 to analyze sports game data and predict results by bivariate Poisson regression model. Karlis also compared prediction results of football and water polo games by bivariate Poisson regression model, prediction results of double independent Poisson distribution and actual situation, and results showed that results of bivariate Poisson regression model were more accurate.

Generally speaking, performance data of two teams in sports games is conventionally analyzed by double independent Poisson distribution. In this way, it is believed that score distribution of two teams is mutually independent and obeys Poisson distribution. However, in fact, score distribution is correlated and scoring ability, pace and home-away environment of one side will have an influence on the other. In particular, during basketball game with fast

pace, pace of one side will inevitably affect the other if scores rise alternately. Therefore, game performance data shall be analyzed and predicted by bivariate Poisson regression model, which has two advantages: firstly, correlation factors in actual situation are included into the model, and secondly, double independent Poisson distribution does not consider about this situation as actually collected data is of excessive variability so that expectation is the same with variance. Bivariate Poisson regression model is featured by excessive variability and thus can better meet requirements of data processing. Thus, the author uses bivariate Poisson regression model to analyze data of regular seasons of 2013-2014 CBA (China Basketball Association) and 2012-2013 NBA (National Basketball Association) respectively, and predict performance in post seasons.

3.1 CBA GAME PERFORMANCE DATA ANALYSIS BY BIVARIATE POISSON REGRESSION MODEL

At present, general point rules of basketball games are 2 scores for winning one game, 1 score for losing one game and 0 score for giving up. Table 1 shows points of each team in 34 rounds of CBA regular season, and specific score data can be seen on official website of China Basketball Association.

TABLE 1 Points of each team in 34 rounds of 2013-2014 CBA regular seasons

Ranking	Team	Winning	Losing	Points	Average scores	Average lost scores
1	Guangdong Winnerway	30	4	64	100.3	88.6
2	Xinjiang Guanghui	26	8	60	104.3	93.5
3	Dongguan Men's Basketball	25	9	59	105.5	100.7
4	Beijing Jinyu	23	11	57	105	98.6
5	Zhejiang Guangsha	21	13	54	108.6	105.7
6	Tianjin Reapal	20	14	54	104.3	103.6
7	Liaoning Hengye	20	14	54	101	99.6
8	Shanghai Men's Basketball	20	14	54	98.5	96.2
9	Shandong Gold	19	15	53	94.6	92.5
10	Fujian Men's Basketball	16	18	50	107.8	108.4
11	Jiangsu China Railway	15	19	49	99.2	100.3
12	Sichuan Aijia	14	20	48	97.5	105.7
13	Zhejiang Chouzhou	13	21	47	106.8	106.8
14	Jilin Rural Commercial Bank	12	22	46	101	105.9
15	Foshan Rural Commercial Bank	11	23	45	100.5	105.8
16	Shanxi Fenjiu	10	24	44	101.5	104.2
17	Bayi Shuanglu	6	28	40	92.7	101.3
18	Qingdao Double Star	5	29	39	102.8	114.6

Assume that points of one side X is X_i , and the other is Y_i , it can be seen from formula (7) that points of each game meet the following distribution:

$$\begin{cases} (X_i, Y_i) \sim P(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}) \quad i = 1, 2, \dots, n \\ \log(\lambda_{1i}) = \mu + h + atth + defg \\ \log(\lambda_{2i}) = \mu + attg + defh \end{cases} \quad (8)$$

where, n is the total number of rounds, μ is a constant, h denotes the influence of home field factors on the game, atth and defh denote attach and defense scores of home team, while attg and defg denote attach and defense scores of away team. In order to make model parameters more identifiable, it is necessary to use standard constraint con-

ditions. Thus, when selecting values of λ_1 and λ_2 , consider that two teams play on a neutral field and have the same physical strength. In addition, parameters of attach and defense shall be those that represent average attach and defense abilities of teams. Parameter λ_3 denotes random factors that affect game results, such as pace and audience atmosphere etc. Thus, covariance λ_{3i} can be expressed as:

$$\log(\lambda_{3i}) = \beta^{con} + \gamma_1 \beta_{hi}^{home} + \gamma_2 \beta_{gi}^{away} \quad (9)$$

where, β^{con} is a constant, β_{hi}^{home} and β_{gi}^{away} denote parameters of home and away field abilities respectively, γ_1 and γ_2 are bivariate indicating parameters with values of 0 or 1

according to actual conditions. If $\gamma_1 = \gamma_2 = 0$, covariance is invariable, and if $\gamma_1 = 1$ and $\gamma_2 = 0$, covariance is only related to home team. Expected scores of two teams are:

$$\begin{cases} E(X_i) = (1-p)(\lambda_{1i} + \lambda_{2i}) + p\theta_1 \\ E(Y_i) = (1-p)(\lambda_{2i} + \lambda_{3i}) + p\theta_1 \end{cases} \quad (10)$$

where, P and θ_1 are estimated mixing ratio and expansion factor respectively. According to statistics of existing game data, the following game performance can be estimated by formula (10). Difficulty of solution-finding by this model lies in estimation of regression coefficient vector P , and commonly used estimation methods include Newton-Raphson Method and Expectation Maximization Algorithm. Thus, this paper will analyze game performance data by these two methods respectively.

Firstly, solve bivariate Poisson regression model of CBA game performance data by Newton-Raphson Algorithm, iterative formula of which is:

$$x^{k+1} = x^k - F'(x^k)^{-1} F(x^k), k=0,1,2,\dots,$$

where $F(x^k)$ is a functional matrix. When solving formula (9) by this iterative formula, take second-order derivative of equation in formula (9). Similarly, from the initial value, estimated value of regression coefficient in the model can be obtained through repeated iteration.

Newton-Raphson Algorithm has fast rate of convergence, but has a high requirement for selection of initial value and requires that the selected initial value shall be as close as possible to accurate value of function, so that results can converge, otherwise, calculation effects cannot be reached. When determining initial value, value estimated under independent conditions is generally selected.

By solving the model, the author analyzes the game results, predicts results of post season, compares with actual situation and shows the results in table 2. It shall be noted that winning-losing relationship between teams is converted into points for the convenience of calculation. For example, team A wins 3 out of 5 rounds with team B, and the point is 8-7.

TABLE 2 Actual and predicted results of 2013-2014 CBA post season (take $\mu=0$)

Round	Both sides	Actual winning-losing	Actual points	Predicted points
1/4 Final	Guangdong Winnerway-Shanghai Men's Basketball	3-0	6-3	5.2-3.4
1/4 Final	Xinjiang Guanghui-Liaoning Hengye	3-1	7-5	6.3-3.9
1/4 Final	Dongguan Men's Basketball-Tianjin Reapal	3-1	7-5	6.1-4.1
1/4 Final	Beijing Jinyu-Zhejiang Guangsha	3-1	7-5	5.9-4.3
Semifinal	Guangdong Winnerway-Beijing Jinyu	2-3	7-8	7.5-6
Semifinal	Xinjiang Guanghui-Dongguan Men's Basketball	3-0	6-3	4.5-3
Final	Xinjiang Guanghui-Beijing Jinyu	2-4	8-10	8.4-7.2

From comparison with actual points, predicted points can predict winning-losing relationship in games well, except deviation of prediction of Beijing Jinyu.

3.2 NBA GAME PERFORMANCE DATA ANALYSIS BY BIVARIATE POISSON REGRESSION MODEL

NBA has 30 teams, each of which shall play 82 games in regular seasons, and thus more data can be accumulated compared with games in CBA. Here, the author uses expectation maximization algorithm to solve bivariate Poisson regression model of NBA game performance data.

Expectation maximization algorithm contains generally two steps: firstly, calculate expectation, namely estimate expected values of unknown parameters and give current parameter estimation. Secondly, maximize expectation, namely re-estimate distribution parameters to achieve maximum data likelihood and give estimated expectation of location variable. Main idea of this algorithm is: assume that $f(\theta|E)$ is a posterior density function of data E and θ , $f(\theta|E,F)$ is a posterior density function of data E and F and θ . These two functions are called observation posterior density and addition posterior density respectively. Besides, take $f(G|\theta,E)$ as a conditional density function under

the situation that θ and data E are determined. Mean value of observation posterior density shall be calculated by expectation maximization method. Then, steps of expectation calculation and maximization are:

$$Q(\theta|\theta^k, E) = \int \log[f(\theta|E, F)]f(G|\theta^k, E)dF, \quad (11)$$

$$Q(\theta^{k+1}|\theta^k, E) = \max Q(\theta|\theta^k, E), \quad (12)$$

where, θ^k and θ^{k+1} are approximate values after k and $k+1$ times of iteration respectively. Use formula (11) and (12) for repeated iteration until $|\theta^{k+1} - \theta^k|$ is small enough. For bivariate Poisson regression model, covariates of parameters are also different, and only iterative method can be used to calculate. In expectation maximization method, firstly, calculate conditional expectation of parameters to be estimated; and then maximize the obtained expectation to complete estimation of regression coefficient.

The author analyzes data of 2012-2013 NBA regular season, and specific score data can be seen on official website of NBA and thus will not be given in detail here. Besides, the author also predicts results of post season, compares actual situation and shows results in table 3.

TABLE 3 Actual and predicted results of 2012-2013 NBA post season (take $\mu = 0$)

Round	Both sides	Actual winning-losing	Actual points	Predicted points
Western 1/4 Final	Thunder-Rockets	4-2	10-8	8.4-5.8
Western 1/4 Final	Spurs-Lakers	4-0	8-4	7.6-4.2
Western 1/4 Final	Nuggets-Warriors	2-4	8-10	8.2-6.9
Western 1/4 Final	Clippers-Grizzlies	2-4	8-10	7.8-8.3
Eastern 1/4 Final	Heat-Bucks	4-0	8-4	7.7-4.1
Eastern 1/4 Final	Knicks-Celtics	4-2	10-8	9.7-7.2
Eastern 1/4 Final	Pacers-Hawks	4-2	10-8	10.2-7.3
Eastern 1/4 Final	Nets-Bulls	3-4	10-11	10.6-10.2
Western Semifinal	Thunder-Grizzlies	1-4	6-9	9.3-7.6
Western Semifinal	Spurs-Warriors	4-2	10-8	10.3-8.6
Eastern Semifinal	Heat-Bulls	4-1	9-6	10.3-7.2
Eastern Semifinal	Pacers-Knicks	4-2	10-8	9.7-6.3
Western Final	Spurs-Grizzlies	4-0	8-4	9.5-8.4
Eastern Final	Heat-Pacers	4-3	11-10	10.6-9.5
Final	Heat-Spurs	4-3	11-10	9.6-10.4

It can be seen from comparison with actual points that predicted results are basically consistent with actual results. From analysis of the above two cases, results of bivariate Poisson regression model by both Newton-Raphson Method and Expectation Maximization Algorithm can be used to analyze basketball game performance data effectively and predict results.

3.3 COMPARISON BETWEEN BIVARIATE POISSON REGRESSION MODEL AND DOUBLE INDEPENDENT POISSON DISTRIBUTION

For analysis of basketball game data, the greatest difference between double independent Poisson distribution and bivariate Poisson regression model lies in lack of consideration of score correlation of both teams, namely covariance λ_{3i} is equal to 0 in the model.

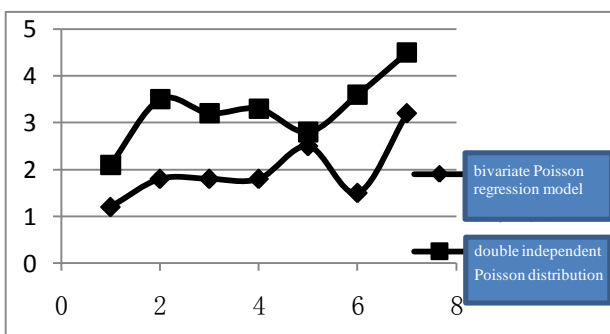


FIGURE 1 Difference between predicted and actual results of bivariate Poisson regression model and double independent Poisson distribution

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Authors



Ke Shen, 16. 4. 1983, China

Current position, grades: A teacher at Physical Education Institute, Hunan University of Technology, China.

University studies: Bachelor Degree of Science in Education in 2005 and Master degree of Science in Education from Central China Normal University, China in 2008.

Scientific interest: Physical education and physical exercise.