

# Supervised orthogonal tensor neighborhood preserving embedding for face recognition

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## Abstract

The deficiency of supervised discriminant information is the problem of Orthogonal Tensor Neighborhood Preserving Embedding (OTNPE) proposed recently for face recognition. So a dimension reduction algorithm called Supervised Orthogonal Tensor Neighborhood Preserving Embedding (SOTNPE) is proposed in the paper. On the basic of OTNPE, the algorithm achieves neighborhood reconstructions within the same class, preserving supervised class label information and neighborhood reconstruction information. Experiments on AR and YaleB face datasets show our proposed algorithm is efficient.

*Keywords:* face recognition, dimensionality reduction, orthogonalization, tensor neighborhood preserving embedding, neighborhood reconstruction

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## 1 Introduction

Manifold learning is an effective way of machine learning in recent years, which discloses geometry structure features hidden in data and has been successfully applied to data mining. Typical manifold dimensionality reduction algorithms include Locally Linear Embedding (LLE) [1], Isometric Feature Mapping (ISOMAP) [2], Laplacian Eigenmaps (LE) [3], Locality Preserving Projection (LPP) [4] and Neighborhood Preserving Embedding (NPE) [5].

NPE is the approximation of local linear embedding, preserving local geometry structure and neighborhood relations. Due to power discriminant performance, NPE has attracted the attention of researchers and has been widely used in face recognition. Nowadays researches on NPE are divided into following three categories according to the processing way of data.

1) Vector based NPE [6-10]. These algorithms need to transform face image matrixes into vectors, which increase complexity of computing matrix to vector conversion.

2) Two-dimensional matrix based NPE [11-13]. However, the algorithms only are limited in the row or column, ignoring the spatial relationship of the image pixels.

3) Second-order and more order tensor based NPE [11-13]. These algorithms represent face image with second-order data, which not only preserve local information of the image pixel but also preserve the spatial structure of the image pixel [14-16]. On the basic of NPE, researchers proposed Tensor Neighborhood Preserving Embedding (TNPE) [15]. Liu et al [17] proposed Orthogonal Tensor Neighborhood Preserving Embedding (OTNPE). By orthogonalizing projections

matrixes, OTNPE has more ability for preserving local geometry structure and neighbor relations and has been successfully applied to facial expression recognition. However, if the sample image data is not smooth and compact in manifold embedded, the discriminant performance of OTNPE is not satisfactory.

Usually supervised information based on class label strengthens the between-class separability of samples, containing discriminant information. Inspired by OTNPE problem, a dimensionality reduction algorithm named Supervised Orthogonal Tensor Neighborhood Preserving Embedding (SOTNPE) for face recognition is proposed in the paper. The algorithm firstly regards multi-dimensional face image as a multi-order tensor data, and then achieves approximately linear reconstruction of samples within the same class and gets projections. Projected data not only preserves the geometric structure and local neighborhood information but also preserves the between-class separability of samples. Experiments on AR and YaleB show that our algorithm is efficient.

The organization of this paper is as follows. Related works is presented in Section 2. We discuss SOTNPE in Section 3. In Section 4, we present experiments for demonstrating the effectiveness of SOTNPE. Conclusions are drawn in Section 5.

## 2 Related Works

### 2.1 NEIGHBORHOOD PRESERVING EMBEDDING (NPE)

Given samples  $X = [x_1, \dots, x_n] \in R^{d \times n}$ , NPE attempts to search the projection matrix  $T$  to get  $Y = T^T X$ . There are some following steps for NPE:

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1) Construct the adjacent graph G. Point sets of G consist of samples and common ways of selecting neighborhoods are  $k$ - nearest neighbors and  $\varepsilon$ -neighbor-hood.

2) Calculate reconstructive weights W. According to the adjacent graph G, each point can be reconstructed through the linear way with its k- neighborhoods. For  $x_i$  in X, the cost of reconstruction of  $x_i$  is described with following function [1]:

$$\begin{aligned} & \min_T \sum_i \left\| x_i - \sum_{j=1}^k w_{ij} x_j \right\|^2 \\ & s.t. \sum_{j=1}^k w_{ij} = 1, \\ & x_j \in O(x_i, k) \end{aligned} \quad (1)$$

where,  $O(x_i, k)$  denotes the k- neighborhoods set of  $x_i$ .

3) Projected data  $Y = T^T X$  satisfies Equation (1), we get:

$$\begin{aligned} & \min_T \sum_i \left\| y_i - \sum_{j=1}^k w_{ij} y_j \right\|^2 = \\ & \min_T \|Y(I-W)\|^2 = \\ & \min_T (Y(I-W)(I-W)^T Y^T) =, \\ & \min_T (T^T X(I-W)(I-W)^T XT) = \\ & \min_T (T^T XMX^T) \end{aligned} \quad (2)$$

where  $M = (I-W)(I-W)^T$ , constrain conditions are introduced as follows:

$$\begin{aligned} & \sum_{i=1}^N y_i = 0 \\ & \frac{1}{N-1} \sum_{i=1}^N y_i^T y_i = I \end{aligned} \quad (3)$$

We replace  $Y = T^T X$  in Equation (2) and Equation (3). The optimization objective function of NPE is listed as follows:

$$\begin{aligned} & \min_T T^T XMX^T \\ & T^T XX^T T = I \end{aligned} \quad (4)$$

## 2.2 TENSOR NEIGHBORHOOD PRESERVING EMBEDDING (TNPE)

TNPE is the tensors extend of NPE. Given  $X = [x_1, \dots, x_n]$  in tensor space  $R^{I_1 \times I_2 \times \dots \times I_k}$ . The destination of TNPE is to search l projection matrixes  $U^i \in R^{m_i \times m_i}$  ( $m_i > m_i, i = 1 \dots l$ ) to preserve local

neighborhood reconstruction. The objective function of TPNE is described as follows:

$$\begin{aligned} & \min \sum_i \left\| x_i \times_1 U^1 \dots \times_k U^k - \sum_j M_{i,j} x_j \times_1 U^1 \dots \times_k U^k \right\|_F^2 \\ & s.t. \sum_i \left\| x_i \times_1 U^1 \dots \times_k U^k \right\|_F^2 = 1 \end{aligned} \quad (5)$$

## 2.3 ORTHOGONAL TENSOR NEIGHBORHOOD PRESERVING EMBEDDING (OTNPE)

On the basic of TNPE, orthogonal conditions of projections matrixes are added in TNPE. The objective function of OTNPE is described as follows:

$$\begin{aligned} & \min \sum_i \left\| x_i \times_1 U^1 \times_2 U^2 \dots \times_k U^k - \sum_j M_{i,j} x_j \times_1 U^1 \times_2 U^2 \dots \times_k U^k \right\|_F^2 \\ & s.t. \sum_i \left\| x_i \times_1 U^1 \times_2 U^2 \dots \times_k U^k \right\|_F^2 = 1 \\ & (U^i)^T U^i = I (i = 1, \dots, l) \end{aligned} \quad (6)$$

## 3 Supervised Orthogonal Tensor Neighborhood Preserving Embedding (SOTNPE)

### 3.1 THE OBJECTIVE FUNCTION

An i-dimensional image itself may be represented as a matrix or a second-order tensor. With tensor algebra used for the analysis of images, an i-dimensional image is regarded as a point of i-order tensor space. On the basic of Equation (6), we have introduced supervision discrimination information based on class label,

Given samples  $X = [x_1, \dots, x_n] = [\chi_1, \dots, \chi_k]$ , where  $\chi_i$  denotes samples of the i-th class. The objective function of SOTNPE is described as follows:

$$\begin{aligned} & \min \sum_i \left\| x_i \times_1 U^1 \times_2 U^2 \dots \times_k U^k - \sum_j M_{i,j} x_j \times_1 U^1 \times_2 U^2 \dots \times_k U^k \right\|_F^2 \\ & s.t. \sum_i \left\| x_i \times_1 U^1 \times_2 U^2 \dots \times_k U^k \right\|_F^2 = 1, \\ & (U^i)^T U^i = I (i = 1, 2, \dots, l) \\ & x_j \in \mathcal{X}_{label(x_i)} \end{aligned} \quad (7)$$

where  $label(x_i)$  denotes the class label of  $x_i$  and  $\mathcal{X}_{label(x_i)}$  denotes samples whose class label is as same as that of  $x_i$ .

### 3.2 ALGORITHM STEPS

**Input:** samples  $X = [x_1, \dots, x_n] \in R^{m_1 \times m_2}$ .

**Output:** projection matrixes  $U^1, U^2, \dots, U^l$ .

**Steps:**

1) On the basic of Equation (1), constrain condition are added, namely that  $y_i$  and  $y_j$  are in the same class.

$$\begin{aligned} & \min_{\mathbf{T}} \sum_i \left\| x_i - \sum_{j=1}^k w_{ij} x_j \right\|^2 \\ & s.t. \sum_{j=1}^k w_{ij} = 1 \\ & x_j \in \mathcal{X}_{label(x_i)} \end{aligned} \quad (8)$$

Calculate the reconstructive matrix  $W$  using the way in [18].

2) Set  $\sum_i U_0^i = I (i=1, \dots, l)$ .

3)

3.1) the number of iterations  $\tau=1, 2, \dots, t$

3.1.1) Calculate 1 projection matrixes in iterations  $m=1, 2, \dots, l$

3.1.1.1) Set  $\Psi_i^k = x_i \times_1 U_\tau^1 \times_2 U_\tau^2 \times \dots \times_l U_\tau^l (i=1, \dots, n)$

3.1.1.2) Set  $D_k = \sum_i \Psi_i^{(k)} (\Psi_i^{(k)})^T$  and

$$S_k = \sum_i \sum_j M_{ij} \Psi_i^{(k)} (\Psi_j^{(k)})^T + \sum_i \sum_j M_{ij} \Psi_j^{(k)} (\Psi_i^{(k)})^T - \sum_i \left( \sum_j M_{ij} \Psi_j^{(k)} \sum_j M_{ij} (\Psi_j^{(k)})^T \right)$$

3.1.1.3) If  $D_k$  is singular, then

$$D_k = D_k + \mu I_{mk} (\mu = 0.001).$$

3.1.1.4) Calculate  $u_i^k$  in  $(D_k - S_k)u_i^k = \lambda_l (D_k)^{-1} u_i^k$  using the generalized matrix solution.

3.1.1.5) loop of iteration  $p=2, \dots, m_k$

3.1.1.5.1) Set

$$U_k^{(E-1)} = [u_1^k, \dots, u_{p-1}^k], H_k^{(p-1)} = [U_k^{(p-1)}]^{-T} (D_k)^{-1} (D_k - S_k)$$

3.1.1.5.2) Set

$$\Phi_k^{(p-1)} = (I_{mk} - (D_k)^{-1} U_k^{(p-1)} [H_k^{(p-1)}]^{-1} [U_k^{(p-1)}]^T) (D_k)^{-1} (D_k - S_k)$$

3.1.1.5.3) Calculate  $u_p^k$  and Orthogonalized

$u_p^k = u_p^k / \|u_p^k\|$  in  $\Phi_k^{(p-1)} u_p^k = \lambda_l I_k u_p^k$  using the generalized matrix solution.

3.1.1.5.4) end the loop p

3.1.1.6) Set  $U_\tau^k = [u_1^k, u_2^k, \dots, u_{m_k}^k]$ .

3.1.1.7) End the loop k.

3.1.2) If  $\tau > 1$  and  $\|U_\tau^k - U_{\tau-1}^k\| < \varepsilon$  ( $\varepsilon$  is error), then

end the loop  $\tau$ .

3.2) End the loop  $\tau$ .

4) Get projection matrixes  $U^i (i=1, 2, \dots, l)$  and

$$Y = X \times_1 U^1 \times_2 U^2 \dots \times_l U^l.$$

**4 Experiments**

**4.1 EXPERIMENTAL DATA**

In the experiment, AR and YaleB face datasets are used as experimental data and they are described as follows:

1) AR [18] consists of over 4000 face images of 126 individuals. For each individual, 26 pictures were taken in two sessions (separated by two weeks) and each section contains 13 images. These images include front view of faces with different expressions, illuminations and occlusions. Figure 1 shows a group of samples in AR.



FIGURE 1 A group of samples in AR

2) YaleB [19] consists of 2414 frontal-face images of 38 individuals. Face images were captured under various laboratory-controlled lighting conditions. Figure 2 shows a group of samples in YaleB.



FIGURE 2 A group of samples in YaleB

**4.2 EXPERIMENTAL SETTINGS**

TPCA [14], TLPP [15], TNPE [15] and OTNPE [17] are selected to compare with our algorithm. Parameter settings of them are listed in Table 1.

TABLE 1 Parameter settings of algorithm

Algorithms	Parameter settings
TPCA	no
TLPP	$\kappa=7$
TNPE	$\kappa=7, l=2$
OTNPE	$\kappa=7, l=2$
SOTNPE	$l=2$

Besides, we randomly select L images per class for training and the remaining for test. Nearest Neighbor algorithm for classification is adopted. All experiments were repeated 40 times and the average of recognition accuracy is gotten as experimental results.

**4.3 EXPERIMENTAL ANALYSIS**

In order to improve the computational efficiency, images of the AR are resized to  $30 \times 30$ . L is set to 6 and 10 and the recognition accuracy is gotten respectively. Figures 3 and 4 show experimental results on AR.

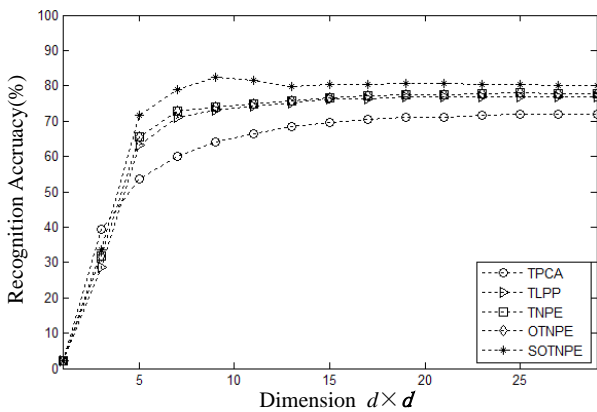


FIGURE 3 Recognition Accuracy VS. Dimension with L=6 on AR

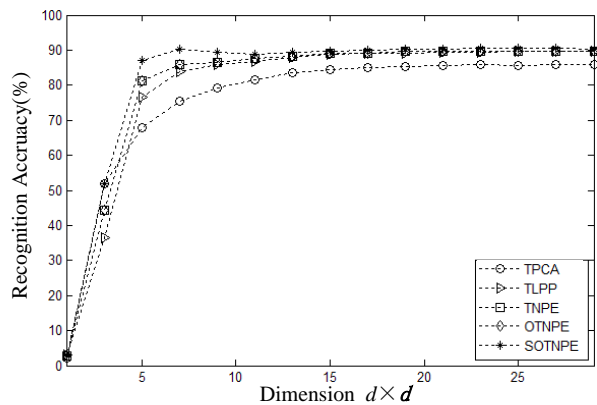


FIGURE 4 Recognition Accuracy VS. Dimension with L=12 on AR

From above Figures 3 and the following conclusions are drawn:

1) With increase in the dimension of the subspace, recognition accuracies of all algorithms improve rapidly. When the dimension exceeds a certain value, the recognition accuracy of TPCA, TLPP, TNPE and OTNPE become gradually stable while that of SOTNPE decreases slowly. This illustrates that SOTNPE gets the best performance in less dimension.

2) In contrast to other algorithms, advantages of SOTNPE decline when the number of training samples L is set to 12, which demonstrates that the overfitting problem exists in SOTNPE.

3) Similarly, images of the YaleB are resized to 30x30. Figures 5 and 6 show experimental results on YaleB.

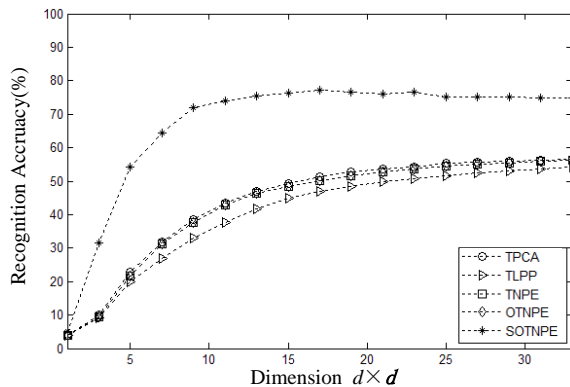


FIGURE 5 Recognition Accuracy VS. Dimension with L=10 on YaleB

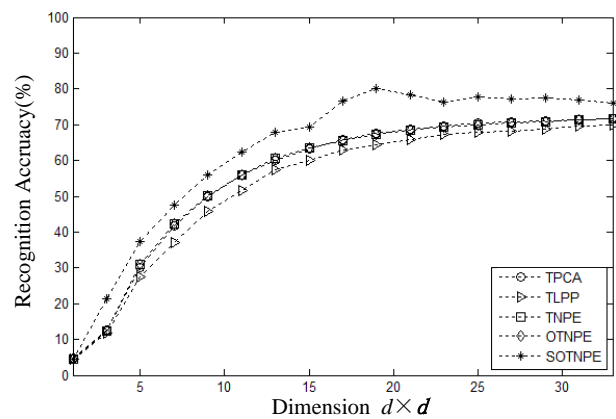


FIGURE 6 Recognition Accuracy VS. Dimension with L=20 on YaleB

We can draw following conclusions from Figure 5 and Figure 6:

1) In contrast to TPCA, TLPP, TNPE and OTNPE, SOTNPE is superior obviously to them. The reason is that SOTNPE not only captures the local nonlinear structure information but also contains discriminant information on YaleB.

2) Advantages of SOTNPE decline when L is set to 20, which also demonstrates that the overfitting problem exists in SOTNPE.

## 5 Conclusions

An algorithm called Supervised Orthogonal Tensor Neighborhood Preserving Embedding (SOTNPE) for dimensionality reduction is proposed in the paper. The algorithm achieves within-class reconstruction instead of reconstruction based on k-nearest neighborhoods of samples on the basic of OTNPE. In contrast to OTNPE, SOTNPE not only inherits the characteristics of OTNPE and fuses more supervision information based on class label, containing power discriminant information. The experiments in the AR and YaleB face database show that SOTNPE outperforms OTNPE obviously. However, like other supervised dimensionality reduction algorithms, the problem of overfitting remains in SOTNPE, how to fuse global unsupervised information is the next work.

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