A new decoupled sliding mode control approach for the linear motion of a spherical rolling robot

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Abstract

This paper investigates the dynamics and control aspects of the linear motion of a pendulum-driven spherical rolling robot. The dynamic model is deduced for the linear motion of a spherical robot by using the Euler-Lagrange formulation. By appropriate definitions the equations of motion for the robotic system are transformed into the state space form. A novel decoupled sliding mode control approach is proposed to achieve set-point regulation of the linear motion. This approach consists of the construction of a cascade sliding mode controller and the design of a nonlinear reaching law by using a switching component that dynamically adapts to the variations of the controlled system. The asymptotic stability of the robotic system is verified through Lyapunov analysis, and the validity of the proposed approach is illustrated through numerical simulations.

Keywords: spherical robot, linear motion, dynamic model, decoupled sliding mode control, exponential reaching law

1 Introduction

A spherical robot is a robotic device without wheels or legs, which has a single spherical form that scrolls by itself to conduct missions. The spherical shape of this class of mobile robots offers several advantages over other forms of surface-based locomotion such as wheels, tracks or legs. The sphere is a strong shape providing a high level of robustness with no major weakness points on its surface, whereas wheels, tracks or legs can be damaged, potentially disabling the mobility of the robot. The outer shell can also be resilient and serve as a protective barrier between the outside environment and the inside equipments. A spherical robot is by nature non-invertible further limiting the risk of becoming disabled, while most other mobile robot designs are vulnerable to tipping over or becoming stuck on the terrain where their means of locomotion lose contact with the ground. These advantages indicate that a spherical robot is appropriate for many different applications such as surveillance, reconnaissance, hazardous environment assessment, search and rescue, as well as planetary exploration.

Spherical rolling robots can be categorized into different types according to their internal driving mechanisms [1-9]. Compared with other types of spherical rolling robots [1-6], a pendulum-driven spherical rolling robot [7-9] has a simpler structure further making it easier to be manoeuvred. The schematic diagram of a pendulum-driven spherical rolling robot with dual inputs is illustrated in Figure 1. Linear motion is a basic form of locomotion of pendulum-driven spherical rolling robots, and it is realized by moving a motor-controlled pendulum forwards or backwards. In this paper, a new decoupled sliding mode control approach based on a novel exponential reaching law is presented for stable control of the linear motion. In the proposed controller, a double layer structure is used to guarantee the stability of the whole system, and the sub-sliding surfaces are utilized to drive the tracking errors to zero.

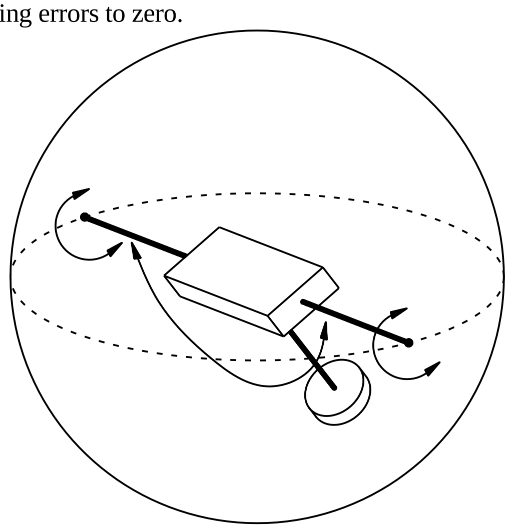

FIGURE 1 Structure of a pendulum-driven spherical robot

2 Dynamic analysis

We start with a simplified planar model, only considering no slip linear motion on flat surfaces. Figure 2 illustrates the simplified model with a side view of a pendulum-driven spherical rolling robot. It represents the spherical shell with its centre of mass *B*, the internal mechanism with its centre of mass *D*, which coincides with that of the spherical shell, and the pendulum (composed of a massless link and a counterweight at its end) with its centre of mass *E* and the axis attached at the centre of the sphere. The definition of the model parameters is listed in Table 1.

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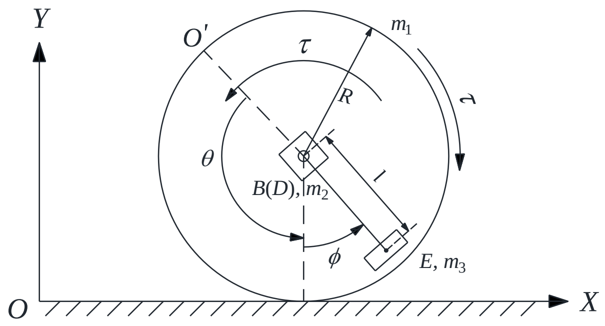

FIGURE 2 Simplified model of the linear motion

TABLE 1 Parameter definition of the planar model

m1, m2, m3	mass of the spherical shell, the internal mechanism and the pendulum, respectively
r, l	radius of the spherical shell and length of the pendulum, respectively
θ, ϕ	roll angle of the spherical shell and sway angle of the pendulum, respectively
I1, I2, I3	moment of inertial of the spherical shell, the internal mechanism and the pendulum, respectively
τ	torque applied to the pendulum

We first choose the roll angle of the sphere θ and the sway angle of the pendulum ϕ as the generalized coordinates of the robotic system, and then we develop the equations of motion by calculating the Lagrangian $L = T - P$ of the system, where T and P are the kinetic energy and potential energy of the system respectively.

The kinetic energy and potential energy of the whole robotic system are given by

$$T = \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}J_2\dot{\phi}^2 + m_3 rl\dot{\theta}\dot{\phi}\cos\phi \quad P = -m_3 gl\cos\phi, \tag{1}$$

where g denotes the gravitational acceleration;

$J_1 = M_t r^2 + I_1$, $M_t = m_1 + m_2 + m_3$, $J_2 = m_3 l^2 + I_2 + I_3$.

It is assumed that the viscous friction operates between the sphere and the pendulum. The loss due to the viscous friction is written in an energy dissipation function that depends on the velocities of the system and the damping coefficient ζ associated with the pendulum-sphere bearing.

$$R = \frac{1}{2}\zeta\left(\dot{\theta} + \dot{\phi}\right)^2. \tag{2}$$

Using the Euler-Lagrange Equations [10], the dynamics of the linear motion can be expressed as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{E}(\boldsymbol{q})\tau, \tag{3}$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the nonlinear terms, and $\boldsymbol{E}(\boldsymbol{q})$ is the input transformation matrix.

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} J_1 & m_3 rl\cos\phi \\ m_3 rl\cos\phi & J_2 \end{bmatrix}, \quad \boldsymbol{E}(\boldsymbol{q}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} \zeta\left(\dot{\theta}+\dot{\phi}\right) - m_3 rl\sin\phi\dot{\phi}^2 \\ \zeta\left(\dot{\theta}+\dot{\phi}\right) + m_3 gl\sin\phi \end{bmatrix}.$$

Using the control input $u = \tau$ and the state vector $\boldsymbol{X} = \left(\theta, \dot{\theta}, \phi, \dot{\phi}\right)^T$, we can rewrite Equation (3) as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(\boldsymbol{X}) + b_1(\boldsymbol{X})u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(\boldsymbol{X}) + b_2(\boldsymbol{X})u \end{cases}, \tag{4}$$

where

$$f_1(\boldsymbol{X}) = \frac{m_{22}n_1 - m_{12}n_2}{m_{12}m_{21} - m_{11}m_{22}}, \quad f_2(\boldsymbol{X}) = \frac{m_{11}n_2 - m_{21}n_1}{m_{12}m_{21} - m_{11}m_{22}},$$

$$b_1(\boldsymbol{X}) = \frac{m_{22} - m_{12}}{m_{11}m_{22} - m_{12}m_{21}}, \quad b_2(\boldsymbol{X}) = \frac{m_{11} - m_{21}}{m_{11}m_{22} - m_{12}m_{21}}.$$

Here m_{ij} denotes the element in the i-th row and j-th column of the matrix $\boldsymbol{M}(\boldsymbol{q})$, and n_k represents the k-th component of the vector $\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})$.

3 Controller design

In this section, we investigate the set-point regulation scheme of the linear motion of a spherical robot, and a decoupled sliding mode controller based on a new exponential reaching law is derived to asymptotically stabilize the robot around its desired equilibrium.

Considering the system represented by Equation (4), we first divide the whole system into two subsystems as $A:\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(\boldsymbol{X}) + b_1(\boldsymbol{X})u \end{cases}$, $B:\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(\boldsymbol{X}) + b_2(\boldsymbol{X})u \end{cases}$.

Then we construct the following first layer sliding surfaces for the two subsystems

$$s_1 = \lambda_1 e_1 + \dot{e}_1, \qquad s_2 = \lambda_2\phi + \dot{\phi}, \tag{5}$$

where λ_1 and λ_2 are positive constants; $e_1 = x_1 - \theta^d$, and θ^d is the desired value of θ.

We define an intermediate variable z which represents the information from subsystem B, and it is incorporated into the sliding surface s_1. Therefore, the second layer sliding surface S is designed as

$$S = \lambda_1(e_1 - z) + \dot{e}_1 \quad z = z_u \cdot \tanh(s_2), \tag{6}$$

where z_u is the upper bound of $abs(z)$, $0 < z_u < 1$; $\tanh(\cdot)$ is the hyperbolic tangent function defined as follows $\tanh(s_2) = \frac{e^{s_2} - e^{-s_2}}{e^{s_2} + e^{-s_2}}$.

Since z_u is less than one, z presents a decaying signal. As s_2 decreases, z decreases too. When $s_2 \to 0$, we have $z \to 0$, $e_1 \to 0$, and then $s_1 \to 0$, and the control objective will be achieved.

Differentiating Equation (6), we can calculate $\dot{z}$ as

$$\dot{z}=\alpha\left(s_2,z_u\right)\cdot\dot{s}_2, \quad (7)$$

where $\alpha\left(s_2,z_u\right)=z_u\cdot\mathrm{sech}^2\left(s_2\right)$, $\mathrm{sech}\left(s_2\right)=\dfrac{2}{e^{s_2}+e^{-s_2}}$.

Differentiating Equation (6) and using Equation (7) yields

$$\begin{aligned}&\dot{S}=\lambda_1\left(\dot{e}_1-\dot{z}\right)+\ddot{e}_1=\\&\lambda_1x_2+f_1-\alpha\lambda_1\left(\lambda_2x_4+f_2\right)+\left(b_1-\alpha\lambda_1b_2\right)u\end{aligned}. \quad (8)$$

Then we can obtain the equivalent control as

$$u_{eq}=\frac{\lambda_1x_2+f_1-\alpha\lambda_1\left(\lambda_2x_4+f_2\right)}{\alpha\lambda_1b_2-b_1}. \quad (9)$$

The control input of the system is assumed to take the following form

$$u=u_{eq}+u_{sw}, \quad (10)$$

where u_{sw} is the switching control.

To construct the switching component u_{sw}, we propose the following exponential reaching law

$$\dot{S}=-\frac{\eta}{N(S)}\mathrm{sgn}\left(S\right)-\rho S, \quad (11)$$

where η and ρ are positive constants $N\left(S\right)=\delta_0+\left(1-\delta_0\right)e^{-\gamma|S|^p}$.

Here δ_0 is a positive constant that is less than one, p is a positive integer, and γ is also a positive constant.

The proposed ERL given by Equation (11) is composed of a variable rate reaching term [12] and an exponential term. Comparing with the conventional exponential reaching law [13], we can see from Equation (11) that if $|S|$ increases, $N\left(S\right)$ approaches δ_0, and therefore $\eta/N\left(S\right)$ converges to η/δ_0, which is larger than η. This means that $\eta/N\left(S\right)$ increases in the reaching phase, and consequently the attraction to the sliding surface S is faster. On the other hand, if $|S|$ decreases, then $N\left(S\right)$ approaches one, and $\eta/N\left(S\right)$ converges to η. This means that, when the system state approaches the sliding surface S, $\eta/N\left(S\right)$ gradually decreases to reduce the chattering. Therefore, the proposed ERL allows the controller to dynamically adapt to the variations of the switching function S by letting $\eta/N\left(S\right)$ vary between η and η/δ_0.

Using Equation (8) to Equation (11), we can obtain the switching control as

$$u_{sw}=\frac{N_\eta\left(S\right)\mathrm{sgn}\left(S\right)+\rho S}{\alpha\lambda_1b_2-b_1}, \quad (12)$$

where $N_\eta\left(S\right)=\dfrac{\eta}{N\left(S\right)}$.

Substituting Equation (12) into Equation (10), we can obtain the following sliding mode control law

$$u=\frac{\lambda_1x_2+f_1-\alpha\lambda_1\left(\lambda_2x_4+f_2\right)+N_\eta\left(S\right)\mathrm{sgn}\left(S\right)+\rho S}{\alpha\lambda_1b_2-b_1}. \quad (13)$$

Theorem 1: Supposing that the robotic system represented by Equation (4) is controlled by the sliding mode controller given by Equation (13). Then the system defined by Equation (4) is asymptotically stable.

Proof: Considering the Lyapunov function candidate $V=\dfrac{1}{2}S^2$, then $\dot{V}$ can be given by

$$\dot{V}=S\dot{S}=-N_\eta\left(S\right)|S|-\rho S^2\le-\eta|S|-\rho S^2\le 0. \quad (14)$$

Integrating both sides of Equation (14), we have

$$\begin{aligned}&V\left(t\right)=\frac{1}{2}S^2\le V\left(0\right)<\infty\\&\lim_{t\to\infty}\int_0^t\left(\eta|S|+\rho S^2\right)\mathrm{d}\sigma\le V\left(0\right)<\infty\end{aligned}. \quad (15)$$

According to Equation (15), we have $S\in L_\infty$, $S\in L_2$. According to Equation (14), we have $\dot{S}\in L_\infty$. Consequently, by applying Babalat's lemma we can conclude that the sliding surface S is asymptotically stable, i.e. $\lim_{t\to\infty}S=0$.

Then the system can be guaranteed to be asymptotically stable.

4 Simulation study

In this simulation, the following physical parameters of the spherical mobile robot [14] and design parameters of the sliding mode controller are used.

$m_1=1.2\,\mathrm{kg}$, $m_2=1.85\,\mathrm{kg}$, $m_3=2.05\,\mathrm{kg}$, $R=0.15\,\mathrm{m}$, $l=0.12\,\mathrm{m}$, $I_1=0.018\,\mathrm{kg\cdot m^2}$, $I_2=0.0017\,\mathrm{kg\cdot m^2}$, $I_3=0.0006\,\mathrm{kg\cdot m^2}$, $\zeta=0.03\,\mathrm{N\cdot m\cdot(rad/s)^{-1}}$, $g=9.81\,\mathrm{m/s^2}$, $z_u=0.96$, $\lambda_1=4$, $\lambda_2=0.6$, $\eta=3$, $\delta_0=0.1$, $\gamma=10$, $p=1$, $\rho=7.8$.

In addition, the initial and desired values of the system states are chosen as $x_0=\left(0,\ 0,\ 0,\ 0\right)^T$, $x^d=\left(\pi,\ 0,\ 0,\ 0\right)^T$.

The simulation results are depicted in Figure 3 to Figure 6. As it is theoretically expected, we can find that both the roll angle of the sphere and the sway angle of the pendulum are asymptotically stabilized to their desired values, and the anti-sway control is achieved in a rapid manner after only one oscillation.

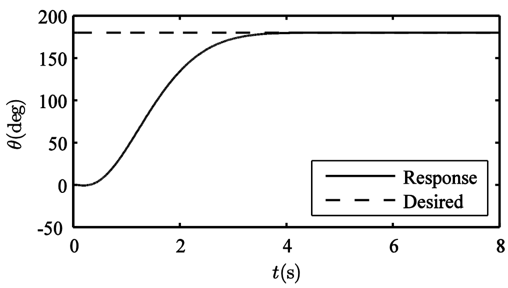

FIGURE 3 Tracking result of the roll angle of the spherical shell

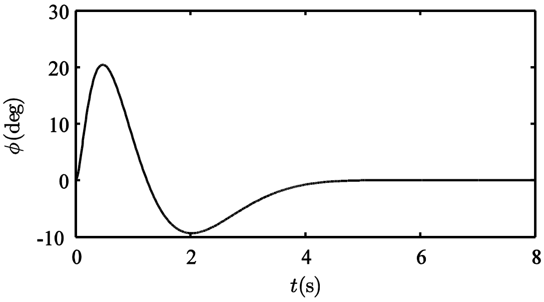

FIGURE 4 Tracking result of the sway angle of the pendulum

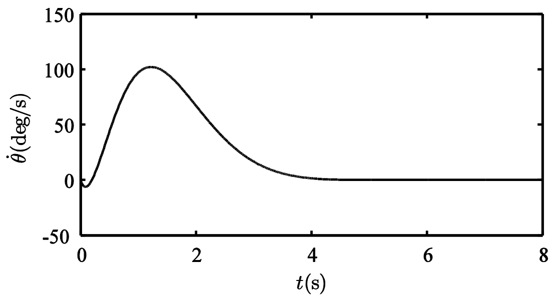

FIGURE 5 Time evolution of the angular rate of the roll angle

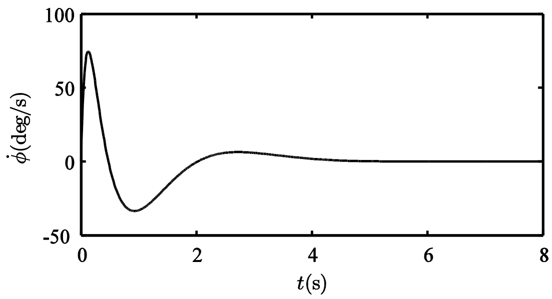

FIGURE 6 Time evolution of the angular rate of the sway angle

5 Conclusions

In this paper, we present a variable structure strategy for set-point regulation of the linear motion of a spherical rolling robot. The control development is based on the construction of a cascade sliding mode controller and a novel exponential reaching law, and the proposed control approach consists of designing a nonlinear reaching law by using a switching term that dynamically adapts to the variations of the system state. The asymptotic stability of the sliding surface of the whole system is theoretically proved, and the simulation results further verify the effectiveness of the proposed controller.

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