Robust model predictive control for descriptor systems with time-delay via dynamic output feedback

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Abstract

The robust model predictive control was investigated for a class of descriptor systems with time-delay and uncertainty, and the dynamic output feedback control law was considered. The systems were transferred to the piecewise continuous descriptor systems and a piecewise constant control sequence was calculated by minimizing a quadratic optimal objective function. At each sampling period, by means of Lyapunov theory and variable transformation, the optimal problem with infinite horizon objective function was reduced to a convex optimization problem involving linear matrix inequalities. The sufficient conditions on the existence of the dynamic output feedback control were derived. Further, an iterative model predictive control algorithm was proposed for the on-line synthesis of dynamic output feedback controllers with the conditions guaranteeing that the closed-loop descriptor systems were regular, impulse-free and robust stable. Finally, a numerical example was presented to show the effectiveness of the proposed approach.

Keywords: Model Predictive Control; descriptor System; Dynamic Output Feedback; Variable Transformation; Linear Matrix Inequality

1 Introduction

Model predictive control (MPC) is a popular strategy, which has been widely adopted in industry as an effective mean of dealing with multivariable constrained control problems and has attracted notable attentions in the control of dynamic systems. MPC requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants, known as the 'time horizon'. Although more than one control move is generally calculated, only the first one is implemented. At the next sampling time, the optimization problem is reformulated and solved with new measurements obtained from the system. Analysis and synthesis approaches for MPC have been extensively studied [1-4].

The class of descriptor (singular) systems has received great interest from mathematic and control theorists. The singular representation, which is a mixture of differential and algebraic equations where the algebraic equations represents the constraints to the solution of the differential part, describes a larger class of systems than the normal linear system model and can be widely seen in large-scale systems, economics, networks, power, neural systems, and others [5, 6]. But the control problems for descriptor systems are much more complicated than that for regular systems because they are required to consider not only stability, but also regular and impulse-free properties. MPC is also essential in the application of descriptor systems. The researches on MPC for descriptor systems has been an field of active researches [7-9]. In Ref [7] a piecewise constant control sequence was calculated by minimizing the worst-case linear quadratic objective function for a class of uncertain descriptor systems. Ref [8] considered the stabilization of continuous time descriptor systems with respect to input constraints and presented a sampled-data model predictive control scheme, and the stability of the closed-loop system was achieved in a similar manner as for non-descriptor systems, utilizing a suitable terminal penalty term and a terminal region constraint. For uncertain descriptor systems with both state and input delays [9], the approximate solutions of optimal problems for infinite time interval and with quadratic performance index were calculated by means of Lyapunov theory and linear matrix inequalities (LMIs) technique, and the sufficient conditions for the existence of the robust predictive control were given in Ref [9].

Most of the existing results of MPC for descriptor systems assume that the states are measurable and the state feedback control laws are implemented, few papers about the output feedback MPC controls for descriptor systems are involved. Static output feedback model predictive control (MPC) is considered for uncertain descriptor systems in Ref. [10]. By Lyapunov theory and relaxation matrices, an iterative MPC algorithm is proposed for the on-line synthesis of static output feedback controllers. Ref. [11] addressed the output feedback robust predictive model control for uncertain descriptor systems

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without time-delay.

The research in this paper is the extensions of the existing results to the more general situations assuming that the states of the plant may be unmeasurable, and the dynamic output feedback control law is considered. The main contribution of this paper is to present the robust MPC for time-delay descriptor systems via dynamic output feedback control, analyse the feasibility of the problem and provide the on-line synthesis of dynamic output feedback controllers guaranteeing the robustness and performance over the whole uncertainty domain.

The paper is organized as follows. A problem formulation and preliminaries on a predictive state-space model as a descriptor system with time-delay and norm-bounded uncertainty are given in the next section. In section 3, by Lyapunov theory and variable transfor- mation, the robust dynamic output feedback control scheme for MPC is worked out. There is an example to illustrate the effectiveness of the proposed method, which is discussed in section 4. Finally, some conclusions are given in the section 5.

Throughout the paper, $\|x\| (x \in \mathbb{R}^n)$ and $\|x\|_Q$ $(Q \in \mathbb{R}^{n\times n})$ denote $\|x\| = (x^{\mathrm{T}}x)^{1/2}$ and $\|x\|_Q = x^T Q x$ respectively. $tr(.)$ denotes the trace of a given matrix. $diag\{\cdots\}$ represents a diagonal matrix. Given a symmetric matrix P, the inequality $P>0$ $(P\geq 0)$ denotes the matrix P positive definiteness (semi-definiteness). The notation $x((k+i)T,kT)$ will be used to define, at time ik-steps ahead, the prediction of a system variable x from time i onwards under a specified initial state and input scenario. I denotes the identity matrix with corresponding dimensions. The symbol * induces a symmetric structure in the matrix.

2 Problem statement and preliminaries

Consider the following descriptor systems with time-delay and norm-bounded uncertainty

$$\begin{cases} E\dot{x}(t) = (A + D_1F(t)H_1)x(t) + A_d x(t-d) \\ \qquad + (B + D_1F(t)H_2)u(t) \\ y(t) = Cx(t),\ \ x(t) = \varphi(t),\ t \in [-d,\ 0] \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the measurement output vector, $\varphi(t)$ is the continuous state initial function, the matrix E may be singular with $\mathrm{rank}(E) = r < n$, d is positive time-delay constants. $E, A, B, C, A_d, D_1, H_1, H_2$ are real constant matrices with appropriate dimensions, and $F(t) \in \mathbb{R}^{m_1\times n_1}$ is a time-varying uncertain matrix function in $\Upsilon = \{F(t) \leq | F(t)^{\mathrm{T}} F(t) \leq I\}$.

Model predictive control for the system (1) will be considered over an infinite horizon. Let T be the fixed sampling interval and $t_{k+1} - t_k = T$. At the sampling time kT for $k = 0,1,2\cdots$, plant measurements are obtained; then a predictive model is used to predict the future behaviours of the system. Let $x(kT+\tau, kT)$ denote the predicted state at time $kT+\tau$, based on the measurements at the sampling time kT, $u(kT+\tau, kT)$ is the control action for time $kT+\tau$ obtained by an optimization problem over the infinite prediction horizon.

For the uncertain descriptor system with time-delay (1), we consider the optimization performance index for robust predictive control in the infinite horizon:

$$\min_{u(kT+\tau),\tau\geq 0} J_\infty(k) \tag{2}$$

$$J_\infty(k) = \max_{\substack{F(kT+\tau)\in\Upsilon,\\ \tau\geq 0}} \int_0^\infty [x^{\mathrm{T}}(kT+\tau)R_1x(kT+\tau) + u^{\mathrm{T}}(kT+\tau)R_2u(kT+\tau)]\mathrm{d}\tau$$

where $R_1 \in \mathbb{R}^{n\times n}$ and $R_2 \in \mathbb{R}^{n\times n}$ are the given weighted matrices. Assume that the system states are not fully measurable, but the system outputs $y(kT)$ are measurable at each sampling period kT.

The purpose of this research is to solve the optimization problem (2) in each sampling period kT, and design a dynamic output feedback controller:

$$\begin{cases} E\dot{\tilde{x}}(kT+\tau) = A_c(k)\tilde{x}(kT+\tau) + B_c(k)y(kT+\tau) \\ u(kT+\tau) = C_c(k)\tilde{x}(kT+\tau),\quad \tau \geq 0 \\ \tilde{x}(0) = x(0) \end{cases}. \tag{3}$$

Meanwhile the close-loop system is regular, impulse-free and asymptotically robust stable, where $\tilde{x}(t) \in \mathbb{R}^n$ is the controller state vector, $A_c(k), B_c(k), C_c(k)$ are the control coefficient matrices to be designed.

Remark 1: $A_c(k), B_c(k), C_c(k)$ remain constant in a certain interval $[kT, (k+1)T)$, but in different intervals, $A_c(k), B_c(k), C_c(k)$ can be different with the change of the time kT. From now on, for clarity, we denote $A_c(k)$, $B_c(k), C_c(k)$ as A_c, B_c, C_c obtained at the sampling time kT.

From the system (1) and the controller (3), the closed-loop descriptor systems is yielded in each sampling period kT.

$$\bar{E}\dot{\bar{x}}(kT+\tau) = (\bar{A} + \bar{D}F(t)\bar{H})\bar{x}(kT+\tau) + \bar{A}_d\bar{x}(kT+\tau-d), \tag{4}$$

where $\bar{E} = \mathrm{diag}\{E, E\}$, $\bar{A} = \begin{bmatrix} A & BC_c \\ B_cC & A_c \end{bmatrix}$, $\bar{D} = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}$, $\bar{H} = \begin{bmatrix} H_1 & H_2C_c \end{bmatrix}$, $\bar{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{x}(kT+\tau) = \begin{bmatrix} x(kT+\tau) \\ \tilde{x}(kT+\tau) \end{bmatrix}$, $\bar{x}(kT+\tau-d) = \begin{bmatrix} x(kT+\tau-d) \\ 0 \end{bmatrix}$.

Substitution of $u(kT+\tau) = C_c\tilde{x}(kT+\tau)$ into performance index (2), the performance index of the closed-loop system (4) is obtained

$$\min_{u(kT+\tau),\tau\geq 0} J_\infty(k)$$

$$J_\infty(k)=\max_{\substack{F(kT+\tau)\in\Upsilon\\ \tau\geq 0}}\int_0^\infty (x^T(kT+\tau)R_1x(kT+\tau)+\tilde{x}^T(kT+\tau)C_c^{\,T}R_2C_c\tilde{x}(kT+\tau))\,\mathrm{d}\,\tau.$$

furthermore, one can have

$$\min_{u(kT+\tau),\tau\geq 0} J_\infty(k)$$

$$J_\infty(k)=\max_{\substack{F(kT+\tau)\in\Upsilon\\ \tau\geq 0}}\int_0^\infty \bar{x}^{\mathrm{T}}(kT+\tau)\begin{bmatrix} R_1 & 0\\ 0 & C_c^{\,\mathrm{T}}R_2C_c\end{bmatrix}\bar{x}(kT+\tau)\,\mathrm{d}\,\tau$$

$$=\max_{\substack{F(kT+\tau)\in\Upsilon\\ \tau\geq 0}}\int_0^\infty \bar{x}^{\mathrm{T}}(kT+\tau)\Phi^{\mathrm{T}}\Phi\bar{x}(kT+\tau)\,\mathrm{d}\,\tau \quad (5)$$

where

$$\bar{x}(kT+\tau)=\begin{bmatrix} x(kT+\tau)\\ \tilde{x}(kT+\tau)\end{bmatrix},\ \Phi=\begin{bmatrix} R_1^{1/2} & 0\\ 0 & R_2^{1/2}C_c\end{bmatrix}.$$

In the receding horizon framework, only the first computed control move $u(kT,kT)$ is implemented. At the next sampling time, the optimization problem (5) is resolved with new measurements from the plant. Now we will review some lemmas for MPC.

Lemma 1 Assume Y,D,E are known real matrices with appropriate dimensions and $Y^T=Y$, for all admissible F satisfying $F^TF<I$, an inequality $Y+DFE+(DFE)^T<0$ holds if and only if there exists a scalar $\varepsilon>0$ such that $Y+\varepsilon DD^T+\varepsilon^{-1}E^TE<0$.

Lemma 2 [9] Descriptor system $E\dot{x}(t)=Ax(t)+A_1x(t-h)$ is regular, impulse-free and asymptotically stable if there exist the matrices Q,P such that

$$E^TP=P^TE\geq 0,$$
$$AP^T+P^TA+P^TA_1Q^{-1}A_1P+Q<0.$$

Lemma 3 [9] Let orthogonal matrices, $U=[U_1\ \ U_2]$, $V=[V_1\ \ V_2]$ be such that $E=U\begin{bmatrix}\Sigma_r & 0\\ 0 & 0\end{bmatrix}V^T$, from which it can be seen that $EV_2=0$, $U_2E=0$, the following items are true.

1. Z satisfying $ZE^T=EZ^T\geq 0$ can be parameterized as $Z=EV_1W^TV_1^T+SV_2^T$ where $W\geq 0\in\mathbb{R}^{r\times r}$, $S\in\mathbb{R}^{n\times(n-r)}$.

2. When $Z=EV_1W^TV_1^T+SV_2^T$ is non-singular and $W>0$, then there exists $\hat{W}$ such that $(EV_1W^TV_1^T+SV_2^T)^{-T}=U_1WU_1^TE+U_2\hat{S}$ with $\hat{W}=\sum_r^{-1}W\sum_r^{-1}$ and $\hat{S}=U_2^T(EV_1W^TV_1^T+SV_2^T)^{-T}$.

3 Output feedback predictive controller design

To solve the robust MPC problem, the key is how to solve the optimization problem (5). One first need to compute $J_\infty(k)$ by a maximization over $F(t)\in\Upsilon$. However, this maximization is not numerically tractable. Hence in Ref. [9], by imposing an inequality constraint, an upper bound for $J_\infty(k)$ is derived, and then the upper bound is minimized.

Consider a Lyapunov-Krasovskii function as follows:

$$V(\bar{x}(t))=\bar{x}(t)^{\mathrm{T}}\bar{E}^{\mathrm{T}}P\bar{x}(t)+\int_{t-d}^{t}\bar{x}(s)^{\mathrm{T}}Q\bar{x}(s)\,\mathrm{d}\,s, \quad (6)$$

where $\bar{E}^{\mathrm{T}}P=P^{\mathrm{T}}\bar{E}\geq 0$ and P is a non-singular, $P,Q\in\mathbb{R}^{2n\times 2n}$, $Q>0$, $\mathrm{rank}(\bar{E})=2r<2n$.

For the robust stability of the system (1), at the sampling time kT, suppose that $V(x(t))$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}\,\tau}\big(V(\bar{x}(kT+\tau,kT))\big)\leq -\bar{x}^{\mathrm{T}}(kT+\tau,kT)\Phi^{\mathrm{T}}\Phi\bar{x}(kT+\tau,kT). \quad (7)$$

For all $F(t)\in\Upsilon$ and $\tau\geq 0$ with the control law (3) and $J_\infty(k)$ to be finite, we must have $x(\infty,kT)=0$ and $V(\infty,kT)=0$ under the control law (3). Hence, by integrating both sides of the aforementioned inequality (7) from $\tau=0$ to ∞, the following is obtained

$$J_\infty(k)\leq V(\bar{x}(kT)). \quad (8)$$

Thus, the robust MPC problem at time kT can be solved by minimizing $V(\bar{x}(kT))$ subject to the imposed the constraint (7).

$$\max_{F(kT)\in\Upsilon} J_\infty(k)\leq V(\bar{x}(kT))\leq\gamma$$

$$V(\bar{x}(kT))=\bar{x}^{\mathrm{T}}(kT)\bar{E}^{\mathrm{T}}P\bar{x}(kT)+\int_{-d}^{0}\bar{x}^{\mathrm{T}}(kT+\tau,kT)Q\bar{x}(kT+\tau,kT)\,\mathrm{d}\,\tau \quad (9)$$

The robust MPC algorithm has been redefined to synthesize, at each time step k, a control law (3) to minimize this upper bound, only the first computed input $u(kT,kT)$ is implemented. At the next sampling time, the output $y((k+1)T)$ is measured, and the optimization is repeated to recomputed the controller $\{A_c(k),B_c(k),C_c(k)\}$.

The following theorem gives the conditions for the feasibility of the optimization problem (5) and the expression of the output feedback controller matrices $\{A_c(k),B_c(k),C_c(k)\}$.

Theorem 1 For the system (1) and the dynamic output feedback controller (3), at the sampling interval $[kT,(k+1)T)$, the optimization problem (5) can be converted to the following optimization problem with the matrix inequality constrains

$$\min\gamma+tr(M_1), \quad (10)$$

$$\text{s.t.}\ \begin{bmatrix}\gamma I & \bar{x}^{\mathrm{T}}(kT)V_1\\ V_1^{\mathrm{T}}\bar{x}(kT) & W_1\end{bmatrix}>0, \quad (11)$$

$$\begin{bmatrix} M_1 & N_1^T\\ N_1 & X\end{bmatrix}>0, \quad (12)$$

$$\begin{bmatrix} \bar{A}Z+Z^{\mathrm{T}}\bar{A}^{\mathrm{T}} & Z^{\mathrm{T}} & Z^{\mathrm{T}}\Phi^{\mathrm{T}} & \bar{A}_d & \bar{D} & Z^{\mathrm{T}}\bar{H}^{\mathrm{T}} \\ Z & -Q^{-1} & 0 & 0 & 0 & 0 \\ \Phi Z & 0 & -I & 0 & 0 & 0 \\ \bar{A}_d^{\mathrm{T}} & 0 & 0 & -Q & 0 & 0 \\ \bar{D}^{\mathrm{T}} & 0 & 0 & 0 & -\xi^{-1}I & 0 \\ \bar{H}Z & 0 & 0 & 0 & 0 & -\xi I \end{bmatrix} < 0, \quad (13)$$

where $\int_{-d}^{0}\bar{x}^{\mathrm{T}}(kT+\tau,kT)\bar{x}(kT+\tau,kT)\mathrm{d}\tau = N_1^T N_1$, W_1, V_1, can be obtained by lemma 3. $Z=P^{-1}$, $X^{-1}=Q>0, M_1$ and scalars $\gamma>0, \xi>0$ are obtained from the optimization problem (10)-(13).

Proof: At the sampling interval $[kT,(k+1)T)$, define a Lyapunov-krasovskii functional as the equation (9):

$$V(\bar{x}(kT)) = \bar{x}^{\mathrm{T}}(kT)\bar{E}^{\mathrm{T}}P\bar{x}(kT) + \int_{-d}^{0}\bar{x}^{\mathrm{T}}(kT+\tau,kT)Q\bar{x}(kT+\tau,kT)\,\mathrm{d}\tau, \quad (14)$$

where $Q>0, E^T P = P^T E \geq 0$ and P is a non-singular matrix.

If there exists a scalar $\gamma>0$ satisfying $\bar{x}^{\mathrm{T}}(kT)\bar{E}^{\mathrm{T}}P\bar{x}(kT)<\gamma$, then $\bar{x}^{\mathrm{T}}(kT)\bar{E}^{\mathrm{T}}P\bar{x}(kT)<\gamma$ is equivalent to the inequality (11) by the Schur complement lemma and lemma 3. Furthermore, an invariant ellipsoid $\chi=\{z \mid z^T V_1 W^{-1} V_1^T z \leq 1\}$ for the predicted states of the uncertain system (1) is obtainned.The second item in the equation (9) may be reduced to

$$\int_{-d}^{0}\bar{x}^{\mathrm{T}}(kT+\tau,kT)Q\bar{x}(kT+\tau,kT)\,\mathrm{d}\tau = \int_{-d}^{0} tr(\bar{x}^{\mathrm{T}}(kT+\tau,kT)X^{-1}\bar{x}(kT+\tau,kT))\,\mathrm{d}\tau = tr(N_1^T N_1 X^{-1}) = tr(N_1^T X^{-1} N_1) \quad (15)$$

Where $X^{-1}=Q$. Assuming there exists a matrix M_1 such that $tr(N^T X_1^{-1} N) < tr(M_1)$, the equation (14) holds by the Schur complement. So $V(x(k)) < \min\ \gamma + tr(M_1)$ and the problem (7) is implied to be $\min\ \gamma + tr(M_1)$.

From the system (4), the derivative of $V_i(x)$ along the inequality (8) can be obtained as follows

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\tau}\left[V(\bar{x}(kT+\tau,kT))\right] \\ &= \dot{\bar{x}}^{\mathrm{T}}(kT+\tau,kT)\bar{E}^{\mathrm{T}}P\bar{x}(kT+\tau,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau,kT)P^{\mathrm{T}}\bar{E}\dot{\bar{x}}(kT+\tau,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau,kT)Q\bar{x}(kT+\tau,kT) \\ &- \bar{x}^{\mathrm{T}}(kT+\tau-d,kT)Q\bar{x}(kT+\tau-d,kT) \\ &= [((\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})\bar{x}(kT+\tau,kT) \\ &\quad + \bar{A}_d\bar{x}(kT+\tau-d,kT))]^{\mathrm{T}}P\bar{x}(kT+\tau,kT) \\ &\quad + \bar{x}^{\mathrm{T}}(kT+\tau,kT)P^{\mathrm{T}}[((\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})\bar{x}(kT+\tau,kT) \\ &\quad + \bar{A}_d\bar{x}(kT+\tau-d,kT))] + \bar{x}^{\mathrm{T}}(kT+\tau,kT)Q\bar{x}(kT+\tau,kT) \\ &\quad - \bar{x}^{\mathrm{T}}(kT+\tau-d,kT)Q\bar{x}(kT+\tau-d,kT) \\ &= \bar{x}^{\mathrm{T}}(kT+\tau,kT)(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})^{\mathrm{T}}P\bar{x}(kT+\tau,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau,kT)P^{\mathrm{T}}(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})\bar{x}(kT+\tau,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau-d,kT)\bar{A}_d^{\mathrm{T}}P\bar{x}(kT+\tau,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau,kT)P^{\mathrm{T}}\bar{A}_d\bar{x}(kT+\tau-d,kT) \\ &+ \bar{x}^{\mathrm{T}}(kT+\tau-d,kT)Q\bar{x}(kT+\tau-d,kT) \\ &\leq -\bar{x}(kT+\tau,kT)^{\mathrm{T}}\Phi^{\mathrm{T}}\Phi\bar{x}(kT+\tau,kT). \end{aligned} \quad (16)$$

The inequality (16) is also equivalent to

$$\begin{bmatrix} \bar{x}(kT+\tau,kT) \\ \bar{x}(kT+\tau-d,kT) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S & P^{\mathrm{T}}\bar{A}_d \\ \bar{A}_d^{\mathrm{T}}P & -Q \end{bmatrix} \begin{bmatrix} \bar{x}(kT+\tau,kT) \\ \bar{x}(kT+\tau-d,kT) \end{bmatrix} \leq 0, \quad (17)$$

where

$$S = (\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})^{\mathrm{T}}P + P^{\mathrm{T}}(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H}) + Q + \Phi^{\mathrm{T}}\Phi.$$

Furthermore, if the following inequality

$$\begin{bmatrix} S & P^{\mathrm{T}}\bar{A}_d \\ \bar{A}_d^{\mathrm{T}}P & -Q \end{bmatrix} < 0 \quad (18)$$

holds, then the inequality (17) holds.

The inequality (18) is implied to be

$$\begin{bmatrix} \bar{A}^{\mathrm{T}}P+P^{\mathrm{T}}\bar{A}+Q+\Phi^{\mathrm{T}}\Phi & P^{\mathrm{T}}\bar{A}_d \\ \bar{A}_d^{\mathrm{T}}P & -Q \end{bmatrix} + \begin{bmatrix} \bar{H}^{\mathrm{T}} \\ 0 \end{bmatrix} F^{\mathrm{T}}(t)\begin{bmatrix} \bar{D}^{\mathrm{T}}P & 0 \end{bmatrix} + \begin{bmatrix} P^{\mathrm{T}}\bar{D} \\ 0 \end{bmatrix} F(t)\begin{bmatrix} \bar{H} & 0 \end{bmatrix} < 0.$$

By lemma 1, there exists a scalar $\xi>0$ such that

$$\begin{bmatrix} \bar{A}^{\mathrm{T}}P+P^{\mathrm{T}}\bar{A}+Q+\Phi^{\mathrm{T}}\Phi & P^{\mathrm{T}}\bar{A}_d & P^{\mathrm{T}}\bar{D} & \bar{H}^{\mathrm{T}} \\ \bar{A}_d^{\mathrm{T}}P & -Q & 0 & 0 \\ \bar{D}^{\mathrm{T}}P & 0 & -\xi^{-1}I & 0 \\ \bar{H} & 0 & 0 & -\xi I \end{bmatrix} < 0.$$

Using the Schur complement lemma, the above inequality can be rewritten as

$$\begin{bmatrix} \bar{A}^{\mathrm{T}}P+P^{\mathrm{T}}\bar{A}+Q & \Phi^{\mathrm{T}} & P^{\mathrm{T}}\bar{A}_d & P^{\mathrm{T}}\bar{D} & \bar{H}^{\mathrm{T}} \\ \Phi & -I & 0 & 0 & 0 \\ \bar{A}_d^{\mathrm{T}}P & 0 & -Q & 0 & 0 \\ \bar{D}^{\mathrm{T}}P & 0 & 0 & -\xi^{-1}I & 0 \\ \bar{H} & 0 & 0 & 0 & -\xi I \end{bmatrix} < 0 \quad (19)$$

Multiplying by $\mathrm{diag}\{P^{-\mathrm{T}}, I, I, I, I\}$ on the left of the inequality (19), by $\mathrm{diag}\{P^{-1}, I, I, I, I\}$ on the right of the inequality (19) respectively, and defining $Z=P^{-1}$, so the inequality (13) holds. The proof of Theorem 1 is completed.

Obviously, the inequalities (11)-(13) with respect to matrices M_1, Z, X, γ, ξ are not LMIs. The inequality (13), the controller coefficient matrices $\{A_c, B_c, C_c\}$ couple with other variables in nonlinear way, so it is difficult to determine the matrices $\{A_c, B_c, C_c\}$ directly from the inequality (13). One thus has a continuous interest to transform the inequality (13) into the LMI form. By the variable transformation method proposed in Ref. [12], a nonlinear matrix inequality can be transformed into a linear matrix inequality, and linear matrix inequality (LMI) can be easily solved by LMI Toolbox in Matlab software.

The matrix Z and its inverse matrix can be divided to the following parts:

$$Z = \begin{bmatrix} \Psi & N \\ N^{\mathrm{T}} & M \end{bmatrix}, \; Z^{-1} = \begin{bmatrix} \Omega & K \\ K^{\mathrm{T}} & L \end{bmatrix}$$

where $\Psi, M, \Omega, L, N = N^T, K = K^T \in \mathbb{R}^{n\times n}$, Ψ, M, Ω, L are non-singular matrices.

From $ZZ^{-1} = I$ and $Z^{-1}Z = I$, it is yielded

$$NK^{\mathrm{T}} + \Psi\Omega = I \text{, } \Omega\Psi + KN^{\mathrm{T}} = I \cdot \tag{20}$$

also it is known that

$$Z\begin{bmatrix} \Omega \\ K^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \Psi & N \\ N^{\mathrm{T}} & M \end{bmatrix}\begin{bmatrix} \Omega \\ K^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

so

$$Z\begin{bmatrix} \Omega & I \\ K^{\mathrm{T}} & 0 \end{bmatrix} = \begin{bmatrix} I & \Psi \\ 0 & N^{\mathrm{T}} \end{bmatrix}.$$

Now, defining

$$F_1 = \begin{bmatrix} \Omega & I \\ K^{\mathrm{T}} & 0 \end{bmatrix}, F_2 = \begin{bmatrix} I & \Psi \\ 0 & N^{\mathrm{T}} \end{bmatrix},$$

one can have

$$ZF_1 = F_2, \;\; F_1^{\mathrm{T}} Z F_1 = F_1^{\mathrm{T}} F_2 = \begin{bmatrix} \Psi^{\mathrm{T}} & I \\ I & \Omega \end{bmatrix}.$$

In order to find out the controller matrices $\{A_c, B_c, C_c\}$, define the following new variables:

$$\begin{cases} \hat{A} = \Omega^{\mathrm{T}} A \Psi + \hat{B} C \Psi + \Omega^{\mathrm{T}} B \hat{C}^{\mathrm{T}} + K A_c N^{\mathrm{T}} \\ \hat{B} = K B_c \\ \hat{C} = N C_c^{\mathrm{T}} \end{cases} . \tag{21}$$

From the definition of the matrices $\{\hat{A}, \hat{B}, \hat{C}\}$ in the equation (21), it is known that for any given non-singular matrices Ω, Ψ and symmetric matrices N, K , the controller matrices $\{A_c, B_c, C_c\}$ can be determined uniquely by solving the equation (21).

Theorem 2 Let $y(kT), \tilde{x}(kT)$ be the output vector of the system (1) and the state vector of the controller (3) measured at the sampling time kT respectively, at the sampling interval $[kT, (k+1)T)$, the controller matrices $\{A_c, B_c, C_c\}$ in the controller (4) are given by the equation (21), where the matrices $\hat{A}, \hat{B}, \hat{C}$, a positive definiteness matrix $Q > 0$, the non-singular matrices Ω, Ψ, X and the scalars $\xi > 0$, $\delta > 0$ are obtained from the following convex optimization problem:

$$\min \gamma + tr(M_1), \tag{22}$$

$$\text{s.t.} \quad \begin{bmatrix} \gamma I & \bar{y}(kT)^{\mathrm{T}} \tilde{C}^{\mathrm{T}} V_1 \\ V_1^{\mathrm{T}} \tilde{C} \bar{y}(kT) & W_1 \end{bmatrix} \ge 0 , \tag{23}$$

$$\begin{bmatrix} M_1 & N_1^{\mathrm{T}} \\ N_1 & X \end{bmatrix} > 0 , \tag{24}$$

$$\begin{bmatrix} \hat{\boldsymbol{J}}_{11} & \boldsymbol{J}_{12} \\ \boldsymbol{J}_{12}^{\mathrm{T}} & \hat{\boldsymbol{J}}_{22} \end{bmatrix} < 0 , \tag{25}$$

$$\begin{bmatrix} X & I \\ I & Q \end{bmatrix} \ge 0 , \tag{26}$$

$$\begin{bmatrix} \delta & 1 \\ 1 & \xi \end{bmatrix} \ge 0 , \tag{27}$$

where

$$\hat{\boldsymbol{J}}_{11} = \begin{bmatrix} \bar{\boldsymbol{J}}_{11} & F_2^{\mathrm{T}} & \tilde{A}_d \\ F_2 & -X & 0 \\ \tilde{A}_d^{\mathrm{T}} & 0 & -Q \end{bmatrix}, \hat{\boldsymbol{J}}_{22} = \mathrm{diag}\{-I, -I, -\delta I, -\xi I\}$$

$$\boldsymbol{J}_{12} = \begin{bmatrix} (R_1^{1/2})^{\mathrm{T}} & 0 & \Omega D_1 & H_1^{\mathrm{T}} \\ \Psi^{\mathrm{T}} (R_1^{1/2})^{\mathrm{T}} & \hat{C} (R_2^{1/2})^{\mathrm{T}} & D_1 & \Psi^{\mathrm{T}} H_1^{\mathrm{T}} + \hat{C} H_2^{\mathrm{T}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\boldsymbol{J}}_{11} = \begin{bmatrix} A^{\mathrm{T}}\Omega + C^{\mathrm{T}}\hat{B}^{\mathrm{T}} + \Omega^{\mathrm{T}} A + \hat{B} C & A^{\mathrm{T}} + \hat{A} \\ A + \hat{A}^{\mathrm{T}} & \Psi^{\mathrm{T}} A^{\mathrm{T}} + \hat{C} B^{\mathrm{T}} + A\Psi + B\hat{C}^{\mathrm{T}} \end{bmatrix},$$

$$\tilde{A}_d = \begin{bmatrix} \Omega^{\mathrm{T}} A_d & 0 \\ A_d & 0 \end{bmatrix}, \; \bar{y}(kT) = \begin{bmatrix} y(kT) \\ \tilde{x}(kT) \end{bmatrix}, \tilde{C} = \begin{bmatrix} (C^{\mathrm{T}}C)^{-1} C^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix}.$$

Proof: At the sampling time kT , it is known that $y(kT) = Cx(kT)$, one can have $C^{\mathrm{T}} y(kT) = C^{\mathrm{T}} C x(kT)$.

Assuming that matrix C is full-column rank, it is obtained:

$$(C^{\mathrm{T}}C)^{-1} C^{\mathrm{T}} y(kT) = x(kT)$$

$$\bar{x}(kT) = \begin{bmatrix} x(kT) \\ \tilde{x}(kT) \end{bmatrix} = \begin{bmatrix} (C^{\mathrm{T}}C)^{-1} C^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} y(kT) \\ \tilde{x}(kT) \end{bmatrix}. \tag{28}$$

Substitution of the equation (28) into the inequality (11), the inequality (11) is equivalent to

$$\begin{bmatrix} \gamma I & \bar{y}(kT)^{\mathrm{T}} \tilde{C}^{\mathrm{T}} V_1 \\ V_1^{\mathrm{T}} \tilde{C} \bar{y}(kT) & W_1 \end{bmatrix} \ge 0 , \tag{29}$$

where

$$\bar{y}(kT)=\begin{bmatrix} y(kT) \\ \tilde{x}(kT) \end{bmatrix},\ \tilde{C}=\begin{bmatrix} (C^{\mathrm{T}}C)^{-1}C^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix}.$$

Multiplying by $\mathrm{diag}\left\{F_1^{\mathrm{T}}, I, I, I, I, I\right\}$ on the left of the inequality (13), by $\mathrm{diag}\left\{F_1, I, I, I, I, I\right\}$ on the right of the inequality (13) respectively, and using the variable transformation (21), it is yielded

$$\begin{bmatrix} \boldsymbol{J}_{11} & \boldsymbol{J}_{12} \\ \boldsymbol{J}_{12}^{\mathrm{T}} & \boldsymbol{J}_{22} \end{bmatrix} < 0\,, \tag{30}$$

where

$$\boldsymbol{J}_{11}=\begin{bmatrix} \bar{\boldsymbol{J}}_{11} & F_2^{\mathrm{T}} & \tilde{A}_d \\ F_2 & -Q^{-1} & 0 \\ \tilde{A}_d^{\mathrm{T}} & 0 & -Q \end{bmatrix},$$

$$\boldsymbol{J}_{22}=\mathrm{diag}\left\{-I,-I,-\xi^{-1}I,-\xi I\right\},\ \tilde{A}_d=\begin{bmatrix} \Omega^{\mathrm{T}}A_d & 0 \\ A_d & 0 \end{bmatrix},$$

$$\bar{\boldsymbol{J}}_{11}=\begin{bmatrix} A^{\mathrm{T}}\Omega+C^{\mathrm{T}}\hat{B}^{\mathrm{T}}+\Omega^{\mathrm{T}}A+\hat{B}C & A^{\mathrm{T}}+\hat{A} \\ A+\hat{A}^{\mathrm{T}} & \Psi^{\mathrm{T}}A^{\mathrm{T}}+\hat{C}B^{\mathrm{T}}+A\Psi+B\hat{C}^{\mathrm{T}} \end{bmatrix},$$

and

$$\boldsymbol{J}_{12}=\begin{bmatrix} (R_1^{1/2})^{\mathrm{T}} & 0 & \Omega D_1 & H_1^{\mathrm{T}} \\ \Psi^{\mathrm{T}}(R_1^{1/2})^{\mathrm{T}} & \hat{C}(R_2^{1/2})^{\mathrm{T}} & D_1 & \Psi^{\mathrm{T}}H_1^{\mathrm{T}}+\hat{C}H_2^{\mathrm{T}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

where

$$\hat{\boldsymbol{J}}_{11}=\begin{bmatrix} \boldsymbol{J}_{11}^{1} & F_1 & \tilde{A}_d \\ F_1^{\mathrm{T}} & -X & 0 \\ \tilde{A}_d^{\mathrm{T}} & 0 & -Q \end{bmatrix},\ \hat{\boldsymbol{J}}_{22}=\mathrm{diag}\left\{-I,-I,-\delta I,-\xi I\right\}.$$

Furthermore,

$$\begin{bmatrix} \hat{\boldsymbol{J}}_{11} & \boldsymbol{J}_{12} \\ \boldsymbol{J}_{12}^{\mathrm{T}} & \hat{\boldsymbol{J}}_{22} \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} X & I \\ I & Q \end{bmatrix} \geq 0, \tag{32}$$

and

$$\begin{bmatrix} \delta & 1 \\ 1 & \xi \end{bmatrix} \geq 0. \tag{33}$$

The proof is completed.

Obviously, inequalities(23)-(27) are LMIs with respect to matrices $\hat{A},\hat{B},\hat{C}$, Ω,Ψ,X , M_1 and scales ξ,δ,γ . The solutions to LMIs (23)–(27) depend only on the current $y(kT)$, $\tilde{x}(kT)$ at the sampling time kT .

Theorem 2 shows that sufficient condition of the existence of the robust MPC output feedback control law can be a LMIs problem.

4 Robust stability analysis

In section 3, the sufficient LMI conditions of the existence of the robust MPC output feedback control law have been given. In this section, we shall study some properties of the derived MPC algorithm, and most importantly, establish the robust stability of the closed-loop system, thus the following results will also be required. It states that if the optimization problem in Theorem 2 is feasible for the sampling time kT , then it is feasible at any sampling time NT with $N>k$.

Lemma 4[13] (Feasibility)**.** Any feasible solution of the optimization (22)-(27) at time kT is also feasible for all times $t>k$, thus, if the optimization problem (22) is feasible at time kT then it is feasible for all times $t>k$.

Theorem 3 If the optimization problems (22)-(27) have the feasible solutions in the moment kT , thus (1) there also exist the feasible solutions in the NT moment $NT(N\geq k)$. (2) We get a piecewise dynamic output feedback control sequence $\{A_c(k),B_c(k),C_c(k)\}_{k=0}^{\infty}$ when k change from 0 to ∞ . Therefore, the closed-loop system, which is composed of piecewise dynamic output feedback control sequence $\{A_c(k),B_c(k),C_c(k)\}_{k=0}^{\infty}$ is regular, impulse-free and asymptotically robust stable.

Proof: First, we will show that the close-loop system is regular and impulse-free, at time interval $t\in[kT,(k+1)T)$, from the definiteness of $V(\bar{x}(kT+\tau,kT))$ and the inequality (7), the following holds

$$\bar{E}^{\mathrm{T}}P=P^{\mathrm{T}}\bar{E}\geq 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\big(V(\bar{x}(kT+\tau,kT))\big)\leq -\bar{x}^{\mathrm{T}}(kT+\tau,kT)\Phi^{\mathrm{T}}\Phi\bar{x}(kT+\tau,kT)$$

where $\Phi>0$, so $\frac{\mathrm{d}}{\mathrm{d}\tau}(V(\bar{x}(t)))<0$ is derived and $V(x(t))$ is strictly decreasing, the close-loop system is asymptotically stable. The inequality (7) is implied to be

$$\begin{bmatrix} \bar{S} & P^{\mathrm{T}}\bar{A}_d \\ \bar{A}_d^{\mathrm{T}}P & -Q \end{bmatrix} < 0\,, \tag{34}$$

where

$$\bar{S}=(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})^{\mathrm{T}}P+P^{\mathrm{T}}(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})+Q$$

Using the Schur complement lemma, the inequality (34) can be written as

$$(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})^{\mathrm{T}}P+P^{\mathrm{T}}(\bar{A}+\bar{D}F(kT+\tau,kT)\bar{H})$$
$$+Q+P^{\mathrm{T}}\bar{A}_dQ^{-1}\bar{A}_d^{\mathrm{T}}P<0$$

By lemma 2, the close-loop system (4) is regular, impulse-free and asymptotically robust stable.

The MPC scheme stated previously is summarized as follows.

Input: the plant (1) and the sampling interval T .

Output: the dynamic output feedback controller matrices $\{A_c,B_c,C_c\}$.

Step1. Let $k=0$

Step2. By lemma 3 and the matrix E in system (1), calculate the matrices W_1, V_1.

Step3. Compute matrix N_1 by $\int_{-d}^{0} \bar{x}^{\mathrm{T}}(kT+\tau,kT)\bar{x}(kT+\tau,kT)\mathrm{d}\tau = N_1^T N_1$, and solve the convex programming problem (22) subject to the LMIs (23)-(27), if there is a feasible solution $(\hat{A},\hat{B},\hat{C},\Omega,\Psi,X,\xi,\delta)$ to the LMIs (23)-(27), then one can compute the equation (21) to obtain the output feedback controller $\{A_c, B_c, C_c\}$.

Step 4. Implement the control action $u(t) = C_c(t)\tilde{x}(t)$ by the controller (3) for $t \in [kT,(k+1)T)$. Meanwhile, measure the outputs $y(kT+T)$ and the controller states $\tilde{x}(kT+T)$ respectively.

Step 5. Let $k = k+1$ and go back to step3.

5 Numerical example

In this section, a numerical example is presented to illustrate the performance of the proposed MPC algorithm for the system (1). Consider an uncertain descriptor system in the form of the system (1), where

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A=\begin{bmatrix}1&1\\0&1\end{bmatrix},\ B=\begin{bmatrix}1&0\\1&2\end{bmatrix},\ C=\begin{bmatrix}1&0.5\\-0.5&1\end{bmatrix},$$

$$A_d=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ D=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},\ H_1=\begin{bmatrix}1&0\\0&0.5\end{bmatrix},$$

$$H_2=\begin{bmatrix}1&1\\0&2\end{bmatrix},\ \bar{y}(0)=\begin{bmatrix}0.5&0&1&0.5\end{bmatrix}^T,$$

$$F(t)=\begin{bmatrix}\sin t&0\\0&e^{-t}\end{bmatrix}$$

where $\Delta^T(t)\Delta(t)\le I$, $h=0.5$, $x(0)=\begin{bmatrix}-1&1.8\end{bmatrix}^T$, $R_1=0.25I$, $R_2=I$ and the sampling interval $T=0.5s$.

By solving the optimization problem given in Theorem 2 via Matlab software, the output trajectories and control inputs trajectories of the descriptor system are shown in FIGURE1 and FIGURE.2. When the time-varying uncertainty is given by $F(t)$. From FIGURE 1 and FIGURE 2, we can observe that the proposed MPC algorithm for the descriptor system works well to asymptotically stabilize the descriptor system.

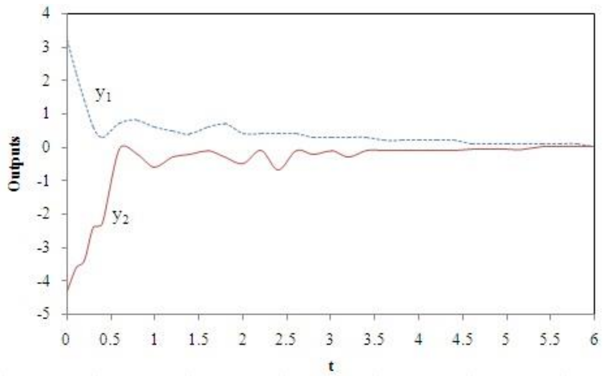

FIGURE 1 Outputs of the close-loop system

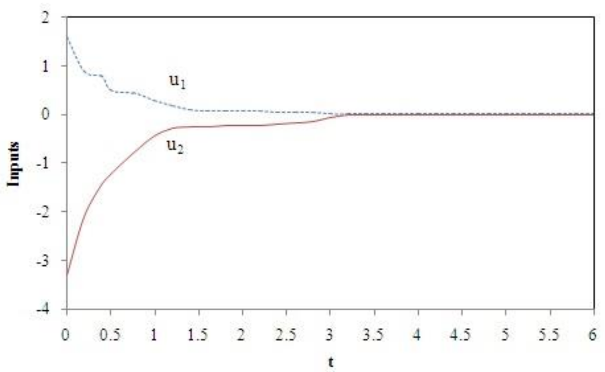

FIGURE 2 Inputs of the close-loop system

6 Conclusion

This paper discussed the dynamic output feedback controllers design method for MPC of a class of uncertain descriptor systems with time delay. The sufficient conditions on the existence of the robust predictive controllers were presented based on Lyapunov stability theory, optimization theory and linear matrix inequality (LMI) method, a parameters notation of output feedback controllers were obtained when these conditions have the feasible solutions. Finally, a numerical example was provided to demonstrate the applicability of the proposed approach.

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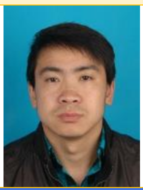

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