Parametric identification for GHM and application of viscoelastic damper

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Received 1 March 2014, www.tsi.lv

Abstract

The GHM (Golla-Hughes-McTavish) model is extensively utilized by structural designers for studying complex structures with viscoelastic damping treatments in engineering. A kind of shear-type viscoelastic damper is investigated, and the damper is modelled with GHM model. The parameters of GHM model are identified by curve fitting and a detailed experiment in complex frequency domain. The comparison results show that the method proposed in the present paper to determine the parameters of GHM model is correct. A whole-spacecraft vibration isolation experiment is practically performed, and the results show that using the method to design the WSVI (Whole-spacecraft Vibration Isolator) is effective for isolating structure vibrations.

Keywords: viscoelastic damper, GHM model, vibration experiment, WSVI

1 Introduction

Among the passive control systems for attenuation of vibrations in structures, high damping properties of the VEM (Viscoelastic Materials) frequently are utilized for vibration isolation [1, 2]. Although the measuring method to dynamic characteristics of VEM has been standardized [3, 4], but the measuring methods of dynamic characteristics of complex structure with the VEM are all not uniform. The theory of sinusoidal sweep and GHM model are combined to measure the dynamic characteristics of the viscoelastic damper in the paper.

The GHM model is presented by Golla and Hughes [5], and improved by McTavish [6]. The GHM model uses a series of small perturbing term to represent the modulus function of the VEM. The complex approach is able to account for damping effects over a range of frequencies and complex mode behaviour. The procedure of GHM starts with plots of experimentally obtained transmissibility in the form of frequency dependent complex moduli that are then curve fit to a rational polynomial over a frequency range of interest. The rational polynomial, with coefficients reflecting the material properties of the test specimen, is used to represent the Laplace transform of the stress-strain relationship. The validation and assessment are addressed with different viscoelastic damping area under consideration. Incorporation of VEM with metal plate can provide an effective means of vibration isolation in the design of whole-spacecraft vibration isolator [7]. The parameters identified provide a conduct for designing the viscoelastic damping vibration isolator.

2 Mathematical model

The constitutive behaviour of VEM might be depended upon the frequency, temperature, amplitude and type of excitation [8]. A mathematical model considering all these effects simultaneously is very difficult to conceive. Therefore, for practical reasons, isothermal conditions are usually considered in the simulation conditions. The frequency dependent constitutive behaviour is taken into account upon the constitutive mathematical model [9].

Considering an isotropic VEM under isothermal and one-dimensional stress conditions, the Boltamann integral constitutive equation [10] of viscoelastic material can be given as:

$$\sigma_{\upsilon}(t)=\int_0^t \dot{g}_{\upsilon}(t-\tau)\varepsilon_{\upsilon}(\tau)d\tau+\varepsilon_{\upsilon}(t)g_{\upsilon}(0)\,, \quad (1)$$

where $\sigma_{\upsilon}(t)$ is the stress of VEM; $\varepsilon_{\upsilon}(t)$ is the corresponding strain; $g_{\upsilon}(t)$ is the relaxation function. Since the fading memory characteristics of VEM, the relaxation function is monotone decreasing function. Considering nil initial conditions, the Laplace transform of the above equation yields:

$$\sigma^*(s)=s\hat{G}(s)\varepsilon(s)=G^*(s)\varepsilon(s)\,, \quad (2)$$

where $\sigma^*(s)$, $\varepsilon(s)$ and $\hat{G}(s)$ are the Laplace transforms of $\sigma(t)$, $\varepsilon(t)$ and $g(t)$ respectively. s is the Laplace complex variable. The GHM model uses a series of mini-oscillator terms to represent the material modulus function in the Laplace domain such that:

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$$s\hat{G}(s) = G_\infty \left[1 + \sum_{r=1}^{N} \alpha_r \frac{s^2 + 2\hat{\zeta}_r \hat{\omega}_r s}{s^2 + 2\hat{\zeta}_r \hat{\omega}_r s + \hat{\omega}_r^2} \right], \quad (3)$$

where the constant G_∞ corresponds to the stable value of the modulus called the final value of the relaxation function. Three positive constants $\{\alpha_r, \hat{\zeta}_r, \hat{\omega}_r\}$ are material parameters determined by curve fitting of the experimental master curves of the VEM, which govern the shape of the modulus function in the Laplace domain.

Consider a single-term GHM, the representation of the complex modulus in the frequency domain is:

$$G^*(\omega) = G_\infty \left[1 + \alpha \frac{\omega^4 + (4\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + 2\hat{\zeta}\hat{\omega}^3 \omega j}{\hat{\omega}^4 + 2(2\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + \omega^4} \right]. \quad (4)$$

There are four constants to be identified to describe the complex mode behaviour of VEM. The stress and strain caused by shear of the VEM can be written as:

$$\tau = \frac{F_s}{A_S}, \ \gamma = \frac{X}{H}, \quad (5)$$

where F_s is external force; A_s is damping area in shear; X is the deformation in shear and H is the thickness of VEM. With the above equations, the complex stiffness is:

$$K_\upsilon^* = \frac{F}{X} = \frac{G^* A_S}{H} = \frac{G_\infty A_S}{H} \left[1 + \alpha \frac{\omega^4 + (4\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + 2\hat{\zeta}\hat{\omega}^3 \omega j}{\hat{\omega}^4 + 2(2\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + \omega^4} \right]. \quad (6)$$

3 The parametric identification of GHM model

The VEM is scarcely used as engineering structure alone due to its low modulus but as sandwich plate with constrained damp layer. Viscoelastic damping material is suitable for sandwich plate with the excellent characteristic of high loss factor, excellent bonding strength with metal.

A kind of shear-type viscoelastic damper is designed within the requirement temperature range considering isothermal conditions. Viscoelastic damping material is pasted with the upper and lower two pieces of metal plate. The dynamic characteristics of viscoelastic damper are analogous with that of the VEM used.

The experiment system, to identify the complex modulus parameters of GHM and the laws of transmissibility versus the frequency, is illustrated in Figure 1.

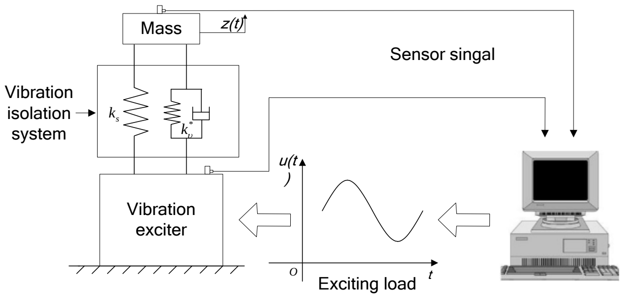

FIGURE 1 Schematic of the experimental system

The system dynamic equation is:

$$m\ddot{z} + k_s(z - u) + k_\upsilon^*(z - u) = 0, \quad (7)$$

where m, k_s and k_υ^* are the mass, stiffness of thin lamina spring and complex stiffness. Excitation displacement and response of isolated structure are:

$$u = Ue^{j\omega t}, \ z = Ze^{j(\omega t + \varphi)}. \quad (8)$$

The transmissibility from the bottom of flange to the top of mass is:

$$|T| = \left| \frac{Z}{U} \right| = \left| \frac{k_s + \frac{G_\infty A_S}{H}\left[1 + \alpha \frac{\omega^4 + (4\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + 2\hat{\zeta}\hat{\omega}^3 \omega j}{\hat{\omega}^4 + 2(2\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + \omega^4} \right]}{k_s - m\omega^2 + \frac{G_\infty A_S}{H}\left[1 + \alpha \frac{\omega^4 + (4\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + 2\hat{\zeta}\hat{\omega}^3 \omega j}{\hat{\omega}^4 + 2(2\hat{\zeta}^2 - 1)\hat{\omega}^2 \omega^2 + \omega^4} \right]} \right|. \quad (9)$$

If the transmissibility data can be obtained, the parameters of GHM model will be identified by curve fitting. The test rig is shown in Figure 2.

FIGURE 2 The experimental set-up for identifying parameters

The upper thin lamina spring provides the restraint of the specimen in pure shear deformation. Electrodynamics' vibration generator provides the dynamic excitation. The acceleration response of the moving mass is measured using a piezoelectric acceleration transducer. One piezoelectric acceleration transducer is located on the flange face as exciting signal and the other is located on the cylindrical core rod which connecting mass blocks and upper plate of the viscoelastic damper. The dynamic frequency responses are all measured in the frequency domain.

The transmissibility from the excitation-point to isolated structure is acquired with sinusoidal excitation. The data are acquired as transmissibility, and complex modulus are extracted from the moduli of the transmissibility. The parameters to identify the GHM model are given: k_s=1.32e^{5}N/m, H=e^{-3}m, m=6.02e^{-1}kg and A_s=4e^{-4}m^2. The comparing results of curve fitting for the transmissibility and the data obtained by practical measurement are illustrated in figure 3. The fitting curve can well represent the experimental measurement results.

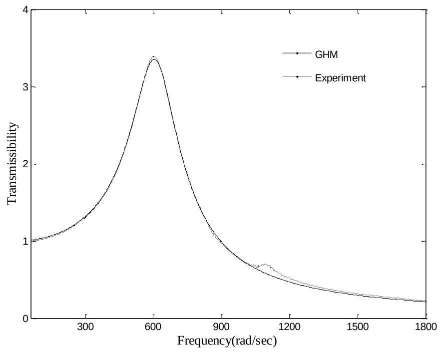

FIGURE 3 Curve fitting of GHM and experimental data (A_s=4e^{-4})

The parameters obtained by above method are shown in table 1. For this material, the parameters were found to represent well the frequency range 10-300Hz.

TABLE 1 The parameters of GHM model

Parameters	G_∞	α	$\hat{\zeta}$	$\hat{\omega}$
Identification	2.132e+05	5.358	13.17	9.989e+04

To verify the parameters, G_∞, α, $\hat{\zeta}$ and $\hat{\omega}$, the experiment with the damping area, A_s=e^{-3}m^2, is performed after plotting curve with the identified parameters in table 1.

In figure 4, the experimental measurement results well reflect the results of GHM model. From figure 3 and figure 4, a conclusion can be drawn that the modulus of viscoelastic damper can be well-fitted using GHM model over a frequency range of interest.

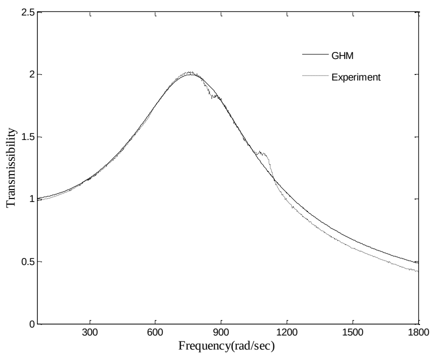

FIGURE 4 Curve fitting of GHM and experimental data (A_s=e^{-3})

4 Application in WSVI

Vibration isolation is an efficient solution for the control of vibrations in structures subjected to viscoelastic damping material. The WSVI system can be modelled as two degrees of freedom, as shown in figure 5.

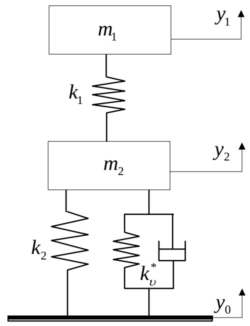

FIGURE 5 Model of WSVI

The dynamic equation of WSVI is:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix}+\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2+k_\upsilon^* \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}=\begin{bmatrix} 0 \\ k_2+k_\upsilon^* \end{bmatrix}\begin{Bmatrix} 0 \\ y_0 \end{Bmatrix}, \quad (10)$$

where m_1 and k_1 are the mass and stiffness of whole-spacecraft, while m_2 and k_2 are the mass and stiffness of vibration isolator.

Excitation displacement, responses of whole-spacecraft and vibration isolator are:

$$y_0 = Y_0 e^{j\omega t},\ y_1 = Y_1 e^{j(\omega t+\psi_1)},\ y_2 = Y_2 e^{j(\omega t+\psi_2)}. \quad (11)$$

The Laplace form of the dynamic equation is:

$$\begin{bmatrix} -m_1\omega^2+k_1 & -k_1 \\ -k_1 & -m_2\omega^2+k_1+k_2+k_\upsilon^* \end{bmatrix}\begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}=\begin{bmatrix} 0 \\ k_2+k_\upsilon^* \end{bmatrix} Y_0. \quad (12)$$

The transmissibility from the bottom of vibration isolator to the mass centre of the whole-spacecraft is:

$$T_1=\frac{Y_2}{Y_0}=\frac{(k_2+k_\upsilon^*)/k_1}{(1-m_1\omega^2/k_1)(1+(k_2+k_\upsilon^*)/k_1-m_2\omega^2/k_1)-1}. \quad (13)$$

A ratio of transmissibility decline, $\delta > 30\%$, is the aim of vibration isolation for the whole-spacecraft. Aiming at δ and using the parameters identified of GHM model, the complex stiffness of viscoelastic dampers can be determined. Then, the damping area and the quantity of the viscoelastic dampers can be well computed. That is the design process of WSVI. The WSVI with the viscoelastic dampers is design to decrease the vibration transmissibility from the bottom of vibration isolator to the mass centre of whole-spacecraft.

FIGURE 6 Experiment set-up of WSVI

The verified experiment is carried out, and the experimental specimen is shown in figure 6. Six viscoelastic dampers, each damping area dimension of which is 0.02×0.05 mm^2, are installed in the WSVI. The thickness of VEM is 0.001m. The mass of whole-spacecraft and the vibration isolator are 40.25kg and 19.35kg. The measurement experiment is driven by sinusoidal signal over a frequency rang of 10-100 Hz.

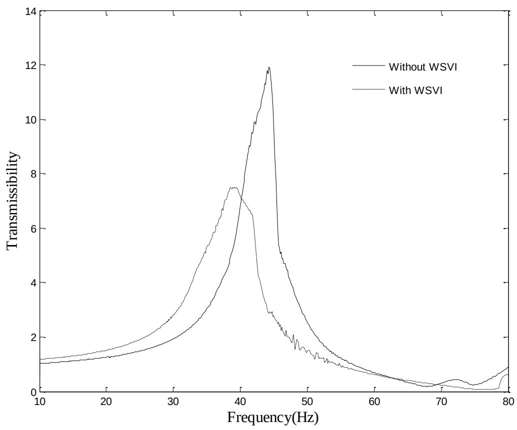

FIGURE 7 Experiment curves of before and after isolation by WSVI

The vibration isolator is connected to the cone-shell adapter. Both the vibration transmissibility from the bottom of platform without vibration isolator to the mass centre of the spacecraft and the vibration transmissibility from the bottom of WSVI to the mass centre of the spacecraft can be acquired. The experiment results are shown in figure 7.

The transmissibility presents great decline at the first-order modal peak after the viscoelastic dampers applied in the vibration system. The transmissibility decreases from 11.87 to 7.449. The ratio of transmissibility decline $\delta = 37.25\%$ exceeds 30%, which can meet the needs of whole-spacecraft vibration isolation.

5 Conclusions

Viscoelastic damping technology is an effective approach to reduce vibration transmitted to whole-spacecraft. An experimental procedure to identify the complex modulus was presented and the obtained data was used to fit the GHM model. The other experimental data was compared in order to validate the experimental procedure and parameters identified for the damping model.

Combing curve fitting and transmissibility experimental data to identification the parameters of GHM model is correct and simple to employ GHM model to perform dynamic analysis. The vibration transmissibility can be decreased signally using the parameters identification method in whole-spacecraft vibration isolation system.

Acknowledgments

The authors gratefully acknowledge the support of the Natural Science Foundation of Heilongjiang Province and the Commission of Science Technology and Industry for National Defense Project, China for this work.

References

[1] Arikoglu A, Ozkol I 2012 *AIAA Journal* **50**(3) 620-30
[2] Meftah S A, Daya E M, Tounsi A 2012 *Thin-Walled Structures* **51** 174-85
[3] Lili C, Baokun H, Tianchang W 2012 *Technical Acoustics* **31**(4) 402-4
[4] Jrad H, Dion J L, Renaud F, Tawfiq I, Haddar M *European Journal of Mechanics-A/Solid* **42** 176-87
[5] Golla D F, Hughes P C 1985 *Journal of Applied Mechanics* **52** 897-907
[6] McTavish D J, Hughes P C, Soucy Y, Graham W B 1992 *AIAA Journal* **30**(5) 1392-9
[7] Shaochong Z, Jinghua L, Jian C, Guoan T 2009 *Journal of Astronautics* **30**(4) 1346-50
[8] Chunhong L, Hongbai B 2007 *Polymer Materials Sscience and Engineering* **23**(6) 28-31
[9] Vasques C M A, Moreira R A S, Rodrigues J D 2010 *Mechanical Engineering* **1**(2) 96-110
[10] Park S W 2001 *International Journal of Solids and Structures* **38**(44) 8065-92

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