Completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata

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Abstract

In order to reduce states of inhomogeneous linear algebraic Hybrid Automaton, the paper proposes completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata. Firstly, it introduces inhomogeneous linear algebraic programs into Hybrid Automata and establishes inhomogeneous linear algebraic Hybrid Automata. And then, it uses mathematical computation and completed trace equivalence to get completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata. Finally, the travel queue automata example shows that completed trace equivalence of inhomogeneous algebraic Hybrid Automata can reduce states.

Keywords: Hybrid Automata, completed trace equivalence, algebraic program

1 Introduction

The main study of Hybrid Systems [1-4] is a class of dynamical systems, which are constituted by continuity subsystems and discrete subsystems. Dynamic characteristics of continuity subsystems evolve with time. Discrete subsystems dynamic evolution is driven by event. Continuity subsystems and discrete subsystems affect mutually. The movement path of Hybrid Systems on the whole shows the transitions of discrete locations. The movement path of Hybrid Systems on the partial shows continual condition approach evolution. Based on the evolution of continuous variable dynamic systems and discrete event systems, it displays a more complex dynamic behaviour. Hybrid Automata are the most commonly used model in Hybrid Systems modelling. They come by finite state machine in theoretical computer science. Hybrid automata models are suitable for the formal verification of Hybrid Systems and security analysis. The formal verification of Hybrid Systems main uses Hybrid Automata models. Hybrid Automata are finite automata with real continuous variables and they can use graphics to describe Hybrid Systems.

Equivalence model is the system simplified model under certain equivalence criteria. The model, which describes system behaviour sometimes, is very complex. In order to understand the interdependences among the main factor or to express and compute model, people always simplify the original model under certain criteria. The equivalence between thing A and thing B generally refers to A and B have common properties in some areas. When people study these common properties, thing A and thing B are the same thing. Completed trace equivalence [5, 6] is a very common equivalence relation and it can optimize model.

This paper is organized as follows. In section 2, it introduces inhomogeneous linear algebraic programs to Hybrid Automata and establishes inhomogeneous linear algebraic Hybrid Automata. In section 3, it uses numerical calculation method and structure of inhomogeneous linear algebraic Hybrid Automata to analyze equivalence of inhomogeneous linear algebraic Hybrid Automata. It gets the completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata by completed trace equivalence theory. In section 4, the travel queue automaton example shows that completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata can optimize inhomogeneous linear algebraic Hybrid Automata.

2 Inhomogeneous linear algebraic Hybrid Automata

If a Hybrid System is divided by time, then discrete event process and continuous variable dynamic process have two different time mode. They are discrete time mode and continuous time mode. The set of discrete times is presented as $t_b \in T_b:\{t_i|i=0,1,\ldots\}$. The set of continuous times is presented as $t_c \in T_c:[t_0,\infty] \subset$ R, R is the set of real numbers. The set of Hybrid Systems times is presented as $T_a=T_b \cup T_c=[t_0,\infty] \subset$ R.

Definition (Inhomogeneous Linear Algebraic Hybrid Automata [7]): an inhomogeneous linear algebraic Hybrid Automaton is a tuple $H=<Q, V, HX, Init, Lab, E, Inv, F, R>$, where:

1) Q is a set of system discrete locations.
2) V is a set of system continuous variables.

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3) *HX* is a set of system continuous variables values.

4) $Init \in Q \times HX$ is a set of system initial states. $Init = \{< q_0, X_0 >\}$. It has only one initial state in the inhomogeneous linear algebraic Hybrid Automata.

5) *Lab* is a set of discrete transition programs. All discrete transition programs are $X' = X$.

6) *E* is a set of discrete transitions.

7) *Inv* is a set of continuous variables invariants.

8) *F* is a set of inhomogeneous linear algebraic programs $X' = AX + b$ which describe system continuous variables dynamic processes.

9) *R* is a set of discrete location transition conditions.

According to the above definition of inhomogeneous linear algebraic Hybrid Automata, the movement process of inhomogeneous linear algebraic Hybrid Automata is as follows:

a) Inhomogeneous linear algebraic Hybrid Automata run the initial state $< q_0, X_0 > \in Init$ as the starting point. The transition processes of inhomogeneous linear algebraic Hybrid Automata include continuous variables dynamic processes and discrete event processes.

b) When the system discrete location is *q*, if system continuous variables value *X* is in the invariant set *Inv*(*q*) of discrete location *q*, then the system carries on continuous evolution. The evolution of continuous variables value *X* follows corresponding inhomogeneous linear algebraic programs. If system continuous variables value *X* is not in the invariant set *Inv*(*q*) of discrete location *q*, then system occurs transition of discrete location.

c) After the discrete location transition, the evolution of continuous variables value *X* follows the new inhomogeneous linear algebraic program. The discrete location is invariable until continuous variables beyond the range of the invariant set.

We can discrete the continuous variables dynamic process of inhomogeneous linear algebraic Hybrid Automata. Continuous time of each discrete location is divided into several same time periods. In each time period, the evolution of continuous variables value *X* follows corresponding inhomogeneous linear algebraic program and the inhomogeneous linear algebraic programs in the same location are same. The elements of continuous variables value *X* in each discrete location are monotonic. Example 1 (Motor speed control automaton): there is a motor speed control automaton and it controls the car acceleration and deceleration by the speed controller. It is designed to control motor speed in 50 kilometers per hour. The rule of speed controller control speed is motor acceleration when motor speed is below 48 kilometers per hour and motor deceleration when speed is above 48 kilometers per hour.

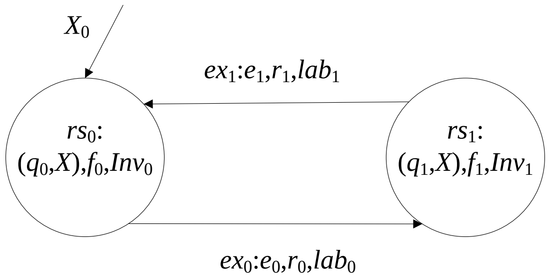

FIGURE 1 Motor speed control automaton

In Figure 1, $X_0 = 48$, initial state is $< q_0, X_0 >$, in the discrete location q_0 motor acceleration, the inhomogeneous linear algebraic program f_0 is $X' = X + 2$ and it expresses motor speed per second increase 2 kilometers per hour, invariant set Inv_0 is $\{X \le 52\}$, discrete transition condition r_0 is *X*=52. In the discrete location q_1 motor deceleration, the inhomogeneous linear algebraic program f_1 is $X' = X - 2$ and it expresses motor speed per second decrease 2 kilometers per hour, invariant set Inv_1 is $\{X \ge 48\}$, discrete transition condition r_1 is *X*=48. The discrete transition program lab_0 and lab_1 are *X*'=*X*.

In the state transition model of inhomogeneous linear algebraic Hybrid Automata, a trace is an action sequence $l_1,l_2,\ldots,l_n$ and satisfies an execute sequence $\eta=s_0l_1,s_1l_2,\ldots,s_nl_n$. $trace(\eta)=$ $l_1,l_2,\ldots,l_n$. An execute sequence $\eta = s_0l_1, s_1l_2, \cdots, s_nl_n$, if $s_0 = s_n$, then this execute sequence constitute a loop. We can be write program $X' = X$ as inhomogeneous linear algebraic program $X' = AX + b$. Let $l_i (i = 1, 2, \cdots, n)$ be $X' = A_i X + b_i$, we can receive:

$$A_1A_2\cdots A_nX + A_1A_2\cdots A_{n-1}b_n + A_1A_2\cdots A_{n-2}b_{n-1} + A_1b_2 + b_1 = X,$$

$$A_1A_2\cdots A_n = E,$$

$$A_1A_2\cdots A_{n-1}b_n + A_1A_2\cdots A_{n-2}b_{n-1} + A_1b_2 + b_1 = 0.$$

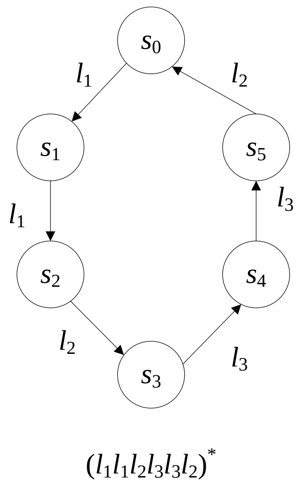

FIGURE 2 State transition model of motor speed control automaton

In Figure 2, it is the state transition model of motor speed control automaton.

3 Completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata

The inhomogeneous linear algebraic Hybrid Automaton has two traces $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$, where $l_i (1 \le i \le m)$ and $h_j (1 \le j \le n)$ respectively be $X' = B_i X + b_i$ and $X' = C_j X + c_j$.

As $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$ have discrete transition relation and continuous transition relation. The equivalence of $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$ not only consider two algebraic programs:

$$X' = B_m B_{m-1} \cdots B_1 X + B_m B_{m-1} \cdots B_2 b_1 + \cdots + B_m b_{m-1} + b_m$$

and

$$X' = C_n C_{n-1} \cdots C_1 X + C_n C_{n-1} \cdots C_2 c_1 + \cdots + C_n c_{n-1} + c_n$$

satisfied mathematical operation condition, but also consider two traces satisfied structure condition. Only when $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$ satisfied mathematical operation condition and structure condition, $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$ are equivalence.

If $B_m B_{m-1} \cdots B_1 = C_n C_{n-1} \cdots C_1$ and $B_m B_{m-1} \cdots B_2 b_1 + \cdots + B_m b_{m-1} + b_m = C_n C_{n-1} \cdots C_2 c_1 + \cdots + C_n c_{n-1} + c_n$, then $l_1, l_2, \cdots, l_m$ and $h_1, h_2, \cdots, h_n$ satisfied mathematical operation condition.

Two traces equivalence structure condition can be divided into 5 types:

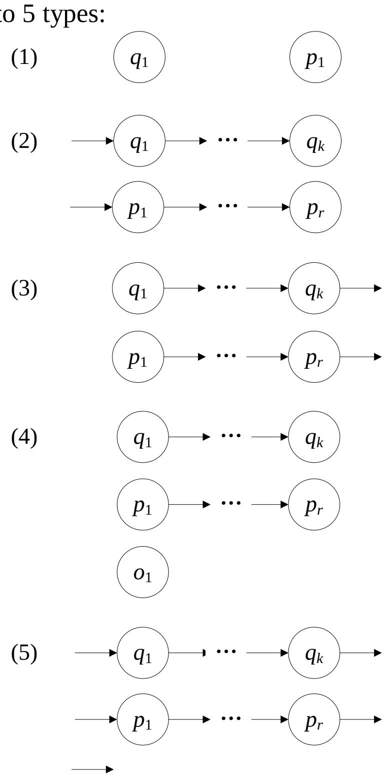

FIGURE 3 Two traces equivalence structure condition

In Figure 3, circles represent continuous variables dynamic process in the discrete locations. Arrows represent discrete transition. In the cases (1-3), two forms satisfied structure condition. In the cases (4) and (5), three forms satisfied structure condition.

We can get the completed trace equivalence automaton of inhomogeneous linear algebraic Hybrid Automata.

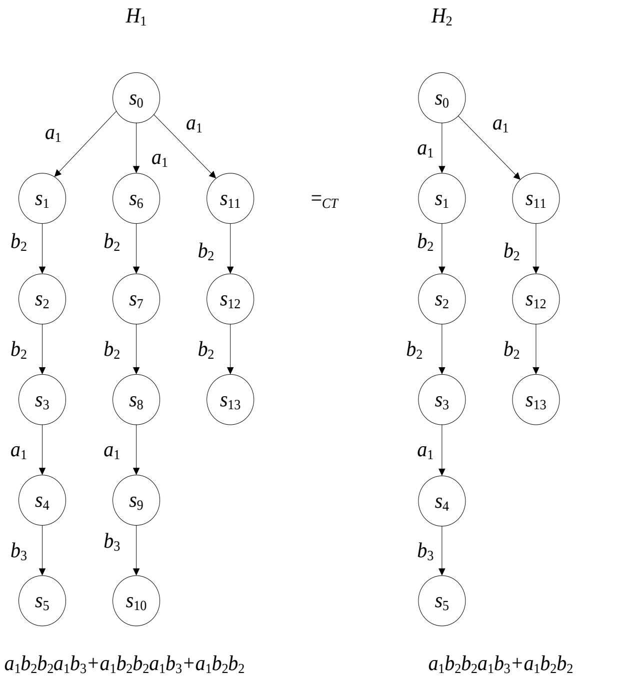

FIGURE 4 An example of completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata state transition model

In Figure 4, H_1 has 14 states and 3 different algebraic programs, H_2 has 9 states and 3 different algebraic programs. H_1 and H_2 have the same function.

Completed trace equivalence of inhomogeneous linear Hybrid Automata can reduce inhomogeneous linear Hybrid Automaton states.

4 Experiments

As the development of the economy, people like travel. The travel queue problem has emerged. We use travel queue automaton example to show the efficient of completed trace equivalence of inhomogeneous linear algebraic Hybrid automata.

The travel queue number is divided into adult number and children number. Many factors affect the travel queue number. We can only consider discrete location duration and weather affect the number. Time period is from 7 am to 10 am. From 7 am to 8 am and from 9 am to 10 am are flat peak period. From 8 am to 9 am is rush peak period.

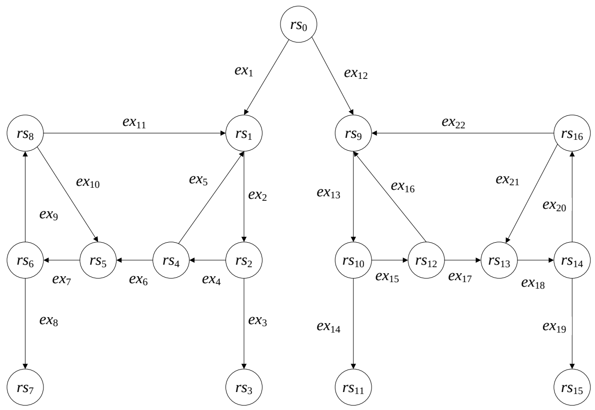

FIGURE 5 Travel queue automaton

The time at 7 am is 0 second. The time from 7 am to 10 am is $t \in [0,10800]$. Adult number and children number respectively are x_1,x_2. $X = (x_1, x_2)^{\mathrm{T}}$ expresses the travel queue number. Let $X_0 = (5,5)^{\mathrm{T}}$. Left side weather is sunny and right side weather is rain. The duration of rs_1, rs_6, rs_9 and rs_{14} is 10 seconds, the duration of rs_2, rs_3, rs_{10} and rs_{11} is 20 seconds, the duration of rs_4, rs_7, rs_{12} and rs_{15} is 30 seconds, the duration of rs_5 and rs_{13} is 5 seconds, the duration of rs_8 and rs_{16} is 45 seconds. One continuous transition time in the $rs_1, rs_2, rs_3, rs_4, rs_5, rs_6, rs_9, rs_{10}, rs_{11}, rs_{12}, rs_{13}, rs_{14}$ is 5 seconds, One continuous transition time in the $rs_7, rs_8, rs_{15}, rs_{16}$ is 15 seconds. The inhomogeneous linear algebraic program:

$$f_1,\ f_2,\ f_9,\ f_{10} \text{ is } X' = \begin{pmatrix} \frac{5}{2} & -1 \\ \frac{1}{2} & 1 \end{pmatrix} X + \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$f_3,\ f_4,\ f_{11},\ f_{12} \text{ is } X' = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{5}{6} \end{pmatrix} X + \begin{pmatrix} -\frac{2}{3} \\ -\frac{2}{3} \end{pmatrix},$$

$$f_5,\ f_6,\ f_{13},\ f_{14} \text{ is } X' = \begin{pmatrix} \frac{23}{4} & -\frac{7}{2} \\ \frac{7}{4} & \frac{1}{2} \end{pmatrix} X + \begin{pmatrix} \frac{5}{2} \\ \frac{5}{2} \end{pmatrix},$$

$$f_7,\ f_8,\ f_{15},\ f_{16} \text{ is } X' = \begin{pmatrix} \frac{1}{18} & \frac{7}{18} \\ -\frac{7}{36} & \frac{23}{36} \end{pmatrix} X + \begin{pmatrix} -\frac{10}{9} \\ -\frac{10}{9} \end{pmatrix}.$$

Invariant set $Inv_1, Inv_5, Inv_9, Inv_{13}$ is:

$\{(5 \le x_1 \le 13.75) \wedge (5 \le x_2 \le 13.75)\}$,

$Inv_2, Inv_3, Inv_6, Inv_7, Inv_{10}, Inv_{11}, Inv_{14}, Inv_{15}$ is:

$\{(13.75 \le x_1 \le 77.734375) \wedge (13.75 \le x_2 \le 77.734375)\}$,

$Inv_4, Inv_8, Inv_{12}, Inv_{16}$ is:

$\{(5 \le x_1 \le 77.734375) \wedge (5 \le x_2 \le 77.734375)\}$.

$f_1 f_1,\ f_2 f_2,\ f_9 f_9,\ f_{10} f_{10},\ f_5,\ f_6,\ f_{13},\ f_{14}$:

are equivalent,

$f_3 f_3,\ f_4 f_4,\ f_{11} f_{11},\ f_{12} f_{12},\ f_7,\ f_8,\ f_{15},\ f_{16}$:

are equivalent.

$f_1 f_1 lab_2 f_2 f_2 f_2 f_2,\ f_5 lab_7 f_6 f_6,\ f_9 f_9 lab_{13} f_{10} f_{10} f_{10} f_{10},$
$f_{13} lab_{18} f_{14} f_{14},\ f_5 f_6 f_6$: are equivalent,

$lab_3 f_3 f_3 f_3 f_3, lab_8 f_7 f_7, lab_{14} f_{11} f_{11} f_{11} f_{11}, lab_{19} f_{15} f_{15}$:

are equivalent,

$f_4 f_4 f_4 f_4 f_4 f_4,\ f_8 f_8 f_8,\ f_{12} f_{12} f_{12} f_{12} f_{12} f_{12},\ f_{16} f_{16} f_{16}$:

are equivalent.

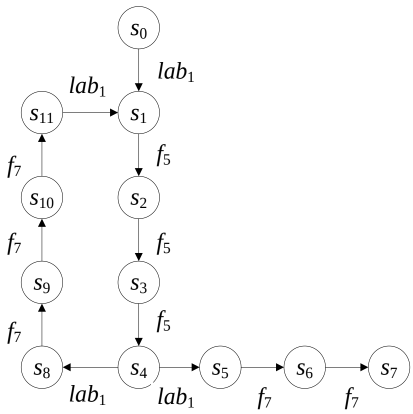

FIGURE 6 Completed trace equivalence state transition model

Using the completed trace equivalence theory, we can get the completed trace equivalence state transition model. The completed trace equivalence automaton of travel queue automaton is shown as Figure 7.

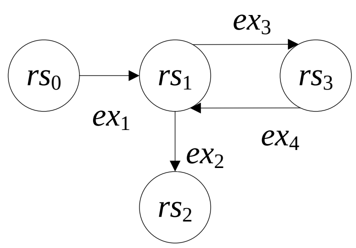

FIGURE 7 Completed trace equivalence automaton of travel queue automaton

From what has been discussion above, completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata can reduce automaton states and optimize inhomogeneous linear algebraic Hybrid Automaton.

5 Conclusion

In this paper, completed trace equivalence of inhomogeneous linear algebraic Hybrid Automata is proposed. It can optimize inhomogeneous linear algebraic Hybrid Automaton and reduce states. In the future work, we will study completed trace equivalence of nonlinear algebraic Hybrid Automata.

Acknowledgments

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