Mechanical characteristics and form-finding analysis of iced transmission lines

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Received 1 October 2014, www.cmnt.lv

Abstract

Form-finding is the important problem to be solved in the cable structure analysis, to the different forms of loads, the direct iterative method is used to determine the initial configuration of the cable structures. Horizontal tension or cable tension is used as iterative convergence condition, the form-finding of the cable is researched under its own gravity, uniform ice and non-uniform ice load. As for the multi-span transmission lines, two conditions of uniform ice and non-uniform ice loads on the whole span were analysed. The results of initial configuration are consistent with the analytical method, which verified the correctness of the direct iterative method, under the condition of non-uniform ice load, the stress of conductor is larger than the maximum stress, which is very dangerous in the actual operation.

Keywords: mechanical characteristics, form-finding, uniform ice, non-uniform ice, finite element method

1 Introduction

In the design of transmission lines, the sag, stress and length of transmission lines are very important parameters, which are the main contents to the mechanics research of transmission lines, this is because the sag and stress directly affect the safety of operation, the small changes and error of cable length will make considerable change to the sag and stress, form-finding is the primary problem to be solved before the analysis and calculation of the cable structures, which is the fundamental prerequisites for dynamic response. Currently, there are four methods for the form-finding analysis of cable structures:

1) The nonlinear finite element method [1,2];

2) The force densities method [3];

3) The dynamic relaxation analysis of form-finding [4, 5];

4) The exact element method [6].

The nonlinear finite element method is most widely used for form-finding. In previous study, the dynamic analysis of cable structures is often assumed to the initial curve with simple shape, the uniform ice and some other non-uniform distributed loads are considered as the ideal uniformly distributed loads, but in many cases, the thickness of ice is not same along the conductors, especially in some micro topography and micro climate area, transmission lines have large elevation difference and big spans, the conductor suffers serious non-uniformly distributed load [7]. Such as the jumping induced by transmission line ice-shedding [8], the research for the dynamic response of ice-shedding on bundled transmission lines [9], and vibration of bundled conductors following ice shedding [10]. The dynamics analysis of the overhead transmission lines with non-uniformly distributed load is accurate based on form-finding, which will be very important to the design of the cable structures.

2 The load distribution of cable structures

The transmission lines belong to one kind of cable structures, the theory of cable structures are based on two assumptions:

1) The cable is flexible: it neither suffers the pressure nor bending;

2) The material of the cable structures follows Hooke's law.

Figure 1 shows an infinitesimal piece of cable, H is horizontal component of the tension T in the tangential direction. V is the vertical component of the T, q_x is the distributed load along the x direction, q_y is the distributed load along the y direction.

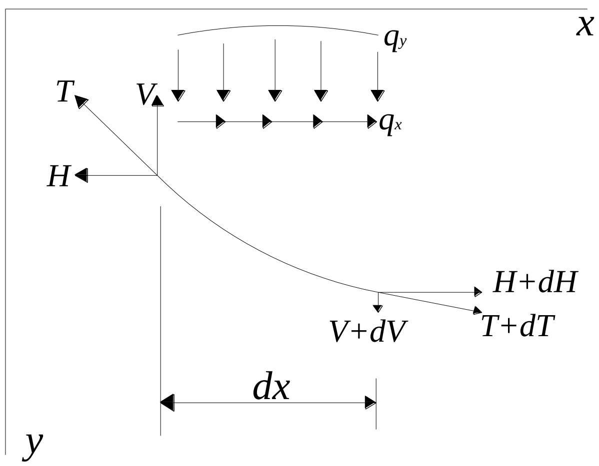

FIGURE 1 Equilibrium of an infinitesimal piece of cable

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The equilibrium equation of the horizontal direction is:

$$\frac{dH}{dx}+q_x=0\,. \tag{1}$$

The equilibrium equation of the vertical direction is:

$$\frac{dV}{dx}+q_y=0\,. \tag{2}$$

As $V=H\dfrac{dy}{dx}$ Equation (1) and (2) can be transformed to:

$$\frac{dH}{dx}+q_x=0\,, \tag{3}$$

$$\frac{d}{dx}\left(H\frac{dy}{dx}\right)+q_y=0\,. \tag{4}$$

Equation (3) and (4) are the governing differential equations of transmission line shape curve. The task for form-finding is to determine the transmission line shape curve, which is to obtain the equilibrium of the applied pretension and external loads by adjusting the form of cable under the given boundary condition.

The uniformly distributed load on the cable structure is generally divided into two forms:

1) uniformly distributed load along chord line of the cable, the shape of cable is parabola;

2) uniformly distributed load along arc length of cable, the shape of cable is catenary, such as the shape of the cable under the action of gravity.

According to the theoretical analysis, the smaller the sag of cable is, the smaller difference of two forms is, to the actual transmission lines, the sag of cable is very small, when the uniformly distributed load is along chord line of the cable, the error of sag is small, it can be received by engineering. The gravity of transmission lines is uniformly distributed along arc length of cable, therefore, under the action of its own gravity, the shape of the transmission line is catenary, rather than a parabola. When the span is small, the sag-span ratio is smaller, the difference is the smaller [11], generally in engineering, if the sag-span ratio is less than 1/8 [12], it can get enough precision of calculating with a parabolic curve.

3 The overall analysis of cable structures

Figure 2 shows a typical joint *J*, Fzie is vertical force of the endpoint i in the element *e*, (*i* changes with different element), *PJ* is the concentrated force acted on the joint *J*.

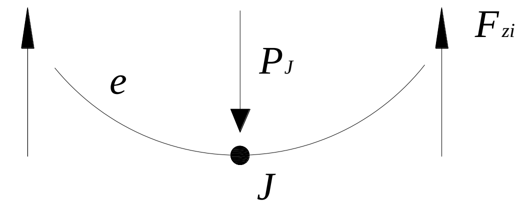

FIGURE 2 Joint J

The vertical equilibrium equation can be written as:

$$\sum_{e_J} F_{zi}^{\ e}-P_J=0\,, \tag{5}$$

where, e_J presents the element connecting with joint *J*, Equation (1) can be written as:

$$\sum_{e_J}\left(k^e_{i1}z^e_{\ 1}+k^e_{i2}z^e_{\ 2}\right)=P_J+\sum_{e_J}P_{Ei}^{\ e}\,. \tag{6}$$

According to the overall integration method, the equation of the global stiffness matrix is:

$$KZ=P\,, \tag{7}$$

where *K* is the global stiffness matrix, *P* is the resultant force from P_J and P_{Ei}^e, *Z* is vector of vertical coordinates.

The state equation of transmission lines is:

$$\sigma_{02}-\frac{E\gamma_2^{\ 2}l^2\cos^3\beta}{24\sigma^2_{\ 02}}=\sigma_{01}-\frac{E\gamma_1^{\ 2}l^2\cos^3\beta}{24\sigma^2_{\ 01}}-\alpha E\left(t_2-t_1\right)\,, \tag{8}$$

σ_{01}, σ_{02} - the stress of transmission lines at the lowest of sag under two kinds of state, MPa;
γ_1, γ_2 - the specific load of transmission lines under two kinds of state;
t_1, t_2 - the temperature of the transmission lines under two kinds of state;
l, β - the span and the angle of differential elevation;
α, E – the temperature expansion coefficient and elastic modulus.

4 The basic principles of form-finding for transmission line

The direct iterative method is used to determine the initial configuration of the cable structure, the basic principles of the direct iterative method is connecting the chord line as the model, using actual material properties and real constants, and set a small initial strain, and then applying the gravity which distributed along the arc length, and gradually update the finite element model, the horizontal tension or cable tension is chose as iterative convergence conditions, the final result is initial deformation of cable structures under its own gravity, the basic process is showed as Figure 3:

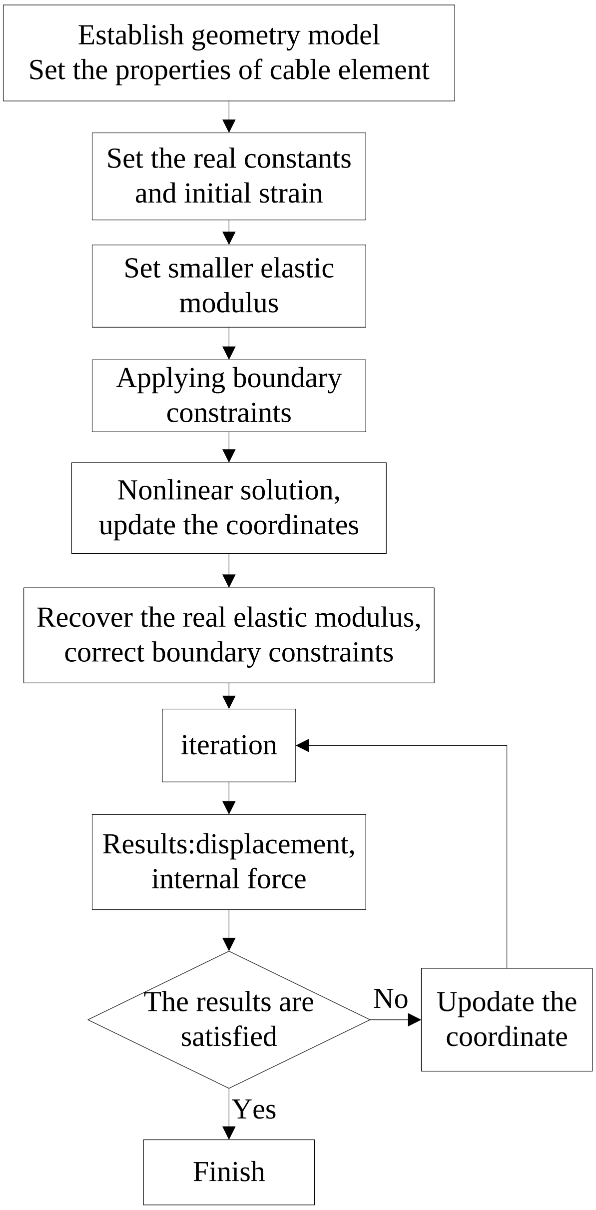

FIGURE 3 The flow chart of form-finding

5 The finite element model of the single span transmission conductor

5.1 THE ESTABLISHMENT OF THE CONDUCTOR MODEL

The finite element method of conductors is transforming the infinite degrees of freedom into a limited degree of freedom. The conductor should be discreted, the stiffness matrix is established through the appropriate shape function of link element, the global stiffness matrix is integrated, the basic equation is established according to the node displacement vector and the node load vector, applying the boundary conditions, the node displacement can be got by solving the basic equations.

The transmission line belongs to a kind of flexible cable structures, which has strong geometric nonlinearity, the geometric characteristics of small strain and large rotation will make stiffness matrix change nonlinearly, the stiffness matrix of structure becomes the function of geometric distortion, the form-finding of transmission lines should open large deformation options in the ANSYS and set the time step, the stress effect should be considered in order to ensure the accuracy of the results.

The form-finding for the transmission lines is analysed by the software ANSYS, the link 10 unit is chose to simulate the lines, the model is connecting a straight line between two points, the straight line is divided into a number of straight link 10 units, the two points are imposed fixed constraints, the gravity loads are imposed along the arc length. The equilibrium configuration of the transmission lines has got under the action of the gravity load, the form-finding of iced transmission lines is based on the equilibrium configuration under gravity. Set nonlinear solution selection. The equilibrium configuration of iced transmission lines is solved, the form-finding of transmission lines is finished under the action of ice load.

Select an overhead transmission line with different height of suspension points, the span is $l = 1000$ m, the elevation difference between the two suspension points is $h = 40$ m, the conductors choose LGJ – 400/35, the technical parameters are shown in Table 1. The ice thickness is 20 mm, the weight per unit length is 15.02 N/m, and the ice density is 0.9 g/cm^3. The mechanical parameters of ice are shown in Table 2.

TABLE 1 the technical parameters of conductor

Number of conductor	Cross-sectional area (mm^2)	Tensile force of calculation (N)	Mass per unit length (kg/km)	allowable stress [σ0] (Mpa)	Diameter (mm)
one	425.24	103900	1349	92.8	26.82

TABLE 2 the mechanical parameters of ice

Variables	Numerical value
The thickness of ice (mm)	20
Density (kg/mm^3)	9×10^{-7}
Poisson's ratio	0.3
The damping ratio	0.1
Coefficient of Comprehensive elasticity (Gpa)	10
Coefficient of Comprehensive expansion (1/°C)	50×10^{-6}

5.2 THE FORM-FINDING OF TRANSMISSION LINES UNDER THE ACTION OF GRAVITY

The conductor is divided into 100 units, the horizontal tension or cable force is the convergence criterion for iteration, the displacement vector diagram of conductor under gravity load as shown in Figure 4, it can be seen that the biggest displacement vector of conductor is 0.654 m, the conductor tends to equilibrium state.

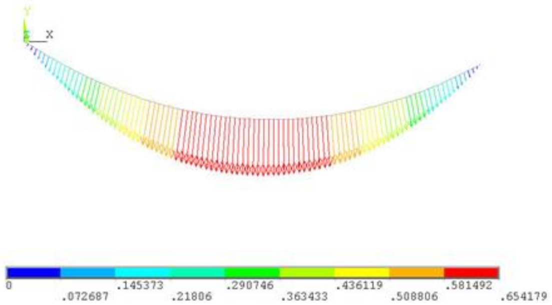

FIGURE 3 The displacement vector diagram of conductor under gravity load (m)

5.3 THE FROM-FINDING OF ICED TRANSMISSION LINES

In many cases, the ice on the conductor is not uniform, it is assumed that the weight is equal with uniform ice and non-uniform ice, the weight of ice on the conductor is measured by the ellipse method or the total weight method, the thickness of uniform ice is 20mm, distributing along the span of the conductors, the ice thickness of different position of is different, which makes the changes of sag and stress are also different, the transmission line is divided into five sections under the action of non-uniform ice load, the horizontal projection length of each section is 200m, the distribution of ice is shown in Table 3.

TABLE 3 The distribution of ice

the case of ice		**the section of ice distribution (mm)**				
		1	*2*	*3*	*4*	*5*
uniform ice		20	20	20	20	20
non-uniform ice	A	30	25	20	15	10
	B	20	30	25	15	10
	C	10	20	30	25	15

Through calculation, the displacement (namely the sag) and stress can be got, analytical values and nonlinear finite element simulation values of representative nodes are listed in Tables 4 and 5 under the condition of self-gravity. It can be seen that the absolute value of the difference between the simulation value with the analytical value is very small, which shows that application of the finite element method to find form of transmission lines is correct and feasible.

TABLE 4 The sag of conductor

		11	**21**	**31**	**41**	**51**	**61**	**71**	**81**	**91**
self-gravity	analytical value	24.73	32.23	37.69	40.97	41.53	41.41	38.13	33.34	26.45
	simulation value	24.82	32.08	37.34	40.60	41.85	41.07	38.25	33.41	26.53
uniform ice	analytical value	25.87	32.74	38.86	41.78	42.94	42.75	39.44	34.35	26.73
	simulation value	25.11	32.63	38.55	41.53	42.86	42.08	39.18	34.15	26.98
the absolute value of the difference	self-gravity	0.09	0.15	0.35	0.37	0.32	0.34	0.12	0.07	0.08
	uniform ice	0.76	0.11	0.31	0.25	0.08	0.67	0.26	0.2	0.25
non-uniform ice	A	25.31	32.92	38.38	41.69	42.88	41.95	38.94	33.86	26.74
	B	25.25	32.92	38.48	41.85	43.04	42.09	39.01	33.90	26.76
	C	25.06	32.64	38.22	41.75	43.14	42.34	39.35	34.21	26.95
	mean value	25.21	32.83	38.36	41.76	43.02	42.13	39.10	33.99	26.82
	Variance	0.016	0.026	0.016	0.006	0.018	0.039	0.049	0.036	0.013

TABLE 5 The stress of conductor

		11	**21**	**31**	**41**	**51**	**61**	**71**	**81**	**91**	**101**
self-gravity	analytical value	57.21	56.64	56.23	55.98	55.92	55.98	56.23	56.64	57.21	57.94
	simulation value	57.19	56.62	56.21	55.97	55.93	55.99	56.24	56.67	57.25	57.99
uniform ice	analytical value	88.4	87.46	86.79	86.38	86.25	86.39	86.80	87.48	88.41	89.6
	simulation value	88.47	87.51	86.82	86.40	86.27	86.50	86.98	87.65	88.56	89.67
the absolute value of the difference	self-gravity	0.015	0.019	0.014	0.009	0.017	0.007	0.012	0.027	0.042	0.05
	uniform ice	0.073	0.047	0.027	0.015	0.017	0.11	0.185	0.172	0.152	0.072
non-uniform ice	A	89.34	88.18	87.43	87.01	86.9	87.07	87.46	88.07	88.85	89.82
	B	91.67	90.68	89.85	89.37	89.25	89.44	89.85	90.47	91.25	92.22
	C	92.61	91.79	91.09	90.62	90.41	90.55	91.0	91.77	92.65	93.74
	mean value	91.21	90.22	89.46	88.99	88.85	89.02	89.44	90.1	90.92	91.93
	Variance	2.83	3.41	3.47	3.37	3.19	3.15	3.26	3.51	3.69	3.89

The sag of analytical and simulation value are shown in Table 4, the absolute value of difference is very small, It illustrates that the finite element method is feasible for simulating from-finding of transmission lines. The sags of iced transmission lines at all the points are larger than bare conductors. It can be seen from that the sag and stress of transmission lines changed with different kinds of the non-uniform ice conditions. Judging from its variance, the change of stress at all points is different with different non-uniform ice conditions, the biggest change appears at both ends and the central span of the conductor.

The deformation diagrams of transmission lines under uniform ice and non-uniform ice are shown in Figure 5-8, the sag and stress have changed a lot, with the increase of elevation difference, the difference of sag and stress becomes larger and larger. The allowable stress of conductor LGJ – 400/35 is 92.8 Mpa, maximum stress does not exceed the allowable stress under uniform ice and condition A, B. But to the condition C, the maximum stress of the iced transmission line at the node 101 is larger than the allowable stress, which will have risk in the actual operation.

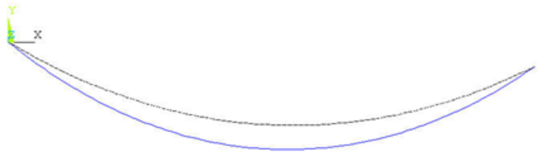

FIGURE 5The deformation diagrams of transmission lines under uniform ice

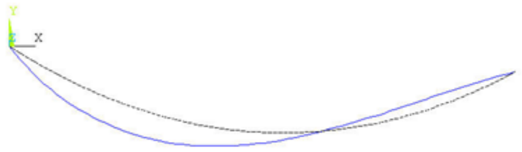

FIGURE 6 The deformation diagrams of transmission lines under ice condition A

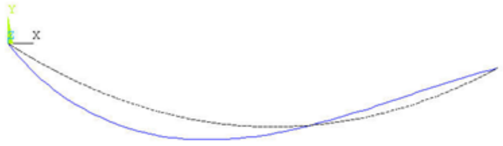

FIGURE 7 The deformation diagrams of transmission lines under ice condition B

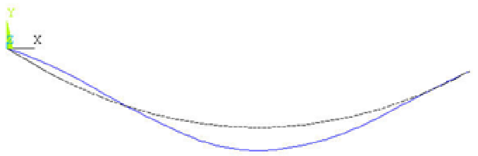

FIGURE 8 The deformation diagrams of transmission lines under ice condition C

6 The form-finding of multi-span transmission lines

6.1 THE MECHANICAL CHARACTERISTICS OF MULTI-SPAN TRANSMISSION LINES

Figure 9 shows the multi-span transmission lines, the force of point A is shown in Figure 10, it does not consider all the friction in the transition zone, it can be obtained the following equilibrium equation without considering the friction.

$$T_1 = T_2 . \tag{9}$$

In general $\theta_1 \neq \theta_2$, therefore:

$$H_1 = T_1 \cos\theta_1 \neq T_2 \cos\theta_2 = H_2 .$$

Applying the nonlinear finite element method to find form of transmission lines, the horizontal tension is the iterative condition (horizontal tension=the average tension stress by the cross-sectional area), for the multi-span transmission lines, the horizontal tension is unequal. The horizontal tension to each adjacent span can be calculated by Equation (9). If the error of horizontal tension is smaller than 5%, update the model with smaller numerical value, or update with a large numerical value.

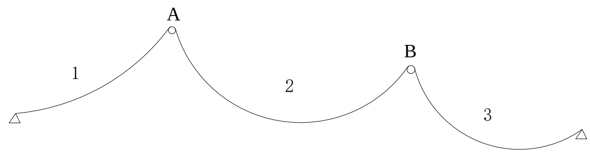

FIGURE 9 The multi-span transmission lines

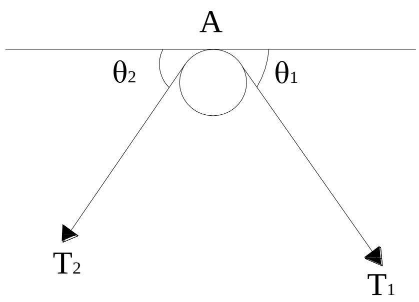

FIGURE 10 The force of point A

6.2 THE EXAMPLE OF FORM-FINDING FOR MULTI-SPAN TRANSMISSION LINES

With reference to the above model parameters of single span transmission line, the model of three consecutive transmission lines was set up, in the actual transmission lines, the adjacent spans are connected by insulators, so the joints of two middle spans are set as hinge constraint, the ends of the lines are used fixed constraint, the Y direction of degrees of freedom at the joints of two middle spans are restrained. Ice conditions of the transmission lines are divided into four kinds of working conditions, which is shown in the Table 6. The thickness of uniform ice is 20 mm. The deformation diagrams of transmission lines under uniform ice and non-uniform ice are shown in Figure 11-14. The distribution of sag has changed a lot with uniform and non-uniform ice, the sag increased in the condition of 2-3, which make the sag of the adjacent span changed, which has a significant impact on the normal operation of transmission lines.

TABLE 6 The ice conditions of three consecutive transmission lines

1	2	3	4
all spans	first span	second span	third span

FIGURE 11 The deformation diagrams of three spans under uniform ice

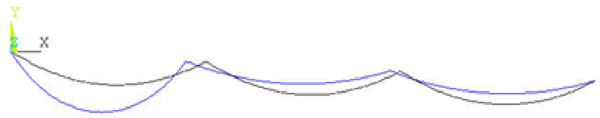

FIGURE 12 The deformation diagrams of the first spans under uniform ice

FIGURE 13 The deformation diagrams of the second spans under uniform ice

FIGURE 14 The deformation diagrams of the third spans under uniform ice

7 Conclusion

1) According to the different forms of load on cable structure, the form-finding of the cable is researched under its own gravity, uniform ice and non-uniform ice load. As for the multi-span transmission lines, two conditions of uniform ice and non-uniform ice loads on the whole span were analysed, which provide premise for the dynamics analysis of transmission lines, the accuracy of form-finding will seriously affect the results of dynamic response.
2) It can be seen that the absolute value of the difference between the simulation and the analytical value is very small, which shows that application of the finite element method to find form of transmission lines is correct and feasible. The sags of iced transmission lines at all the points are bigger than bare conductors. For the non-uniform ice conditions of A, B and C, it can be seen from that the sag and stress of transmission lines changed with different kinds of the non-uniform ice conditions. Judging from its variance, the change of stress at all points is different with different non-uniform ice conditions, the biggest change appears at both ends and the central span of the conductor.
3) The sag and stress have changed a lot between the conditions of uniform ice and non-uniform ice, with the increase of elevation difference, the difference of the sag and stress becomes larger and larger. The allowable stress of conductor LGJ – 400/35 is 92.8 Mpa, the maximum of the iced transmission line stress does not exceed the allowable stress under uniform ice and condition A, B. But to the condition C, the maximum stress at the node. 101 MPa is larger than the allowable stress, which will have risk in the actual operation.

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