Optimal routing strategy on weighted networks

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Abstract

It is of great importance to improve the transfer capacity of the weighted networks. In this paper, the traffic dynamics on weighted networks is investigated based on global information. It is shown by simulations that the weighted network transfer capacity depends strongly on the tuneable parameter in three different node delivery capability schemes: constant, proportional to node degree and proportional to node strength. Furthermore simulations on both computer-generated and real world networks show that different tuneable parameter is suitable for different node delivery capability scheme.

Keywords: weighted network, BBV network, routing strategy, transfer capacity

1 Introduction

Due to the constantly growing significance of large communication networks such as the World-Wide-Web and the Internet, the study of network transfer capacity is becoming increasingly important in the past few years. These real world networks can be properly described as complex networks while nodes representing individuals and edges representing the interactions among them. Consequently, the fast-developing theory of complex networks throw light on how to improve the network transfer capacity since the seminal work on the small-world phenomenon [1] and the scale free property [2]. The previous studies have been primarily focused on unweight networks where edges between nodes are either present or not, represented as binary states. However, lots of real world networks present different strength of the edges between nodes such as the mobile communication networks [3], the scientific collaboration networks [4], the world-wide airport network [5] and the Internet [6]. Networks are specified not only by the topology but also by the weight of edges.

Recently, finding optimal routing strategies to control traffic congestion and improve transfer capacity on a large growing communication network is gaining increasing concern. The shortest path routing strategy [7], where the packets are forwarded following the shortest path, is most commonly adapted because packets may reach their destinations quicker than following other paths. However, if all packets follow the shortest path, it will easily lead to the overload of the high-degree nodes and result in traffic congestion. An effective routing strategy [8] is proposed to redistribute traffic load in central nodes to other non-central nodes which can enhance the network capability in processing traffic more than 10 times. Based on the idea, we have put forward a novel routing strategy for BBV weighted network in which packets are transferred through the path based on the weight of edges with a tuneable parameter α [9]. And different optimal tuneable parameter is suitable to maximize the overall network transfer capacity for different node delivery capability schemes. How to the node characteristics to enhance the transfer capacity deserves special attention.

This paper is organized as follows. In section 2 we describe the BBV weighted network model, the traffic dynamics model and our routing strategy, followed by the experimental evaluations on computer generated networks and real world network in section 3. The conclusions are given in section 4.

2 Models

In those models presented to describe the real world networks [10-12], the BBV model [11] is most widely used. The BBV weighted network can be completely described by an adjacency matrix W, whose elements w_{ij} denote the weight of the edge between node i and j. The definition of the BBV weighted network [11] is based on two coupled mechanisms:

(i) Growth. Starting from an initial small number of N_0 nodes connected by edges with assigned weight w_0, a new node is added at every time step. The new added node is connected to m different previously existing nodes with equal weight w_0 for every edge and chooses preferentially nodes with large strength according to the probability $\prod_{n\to i} = s_i \Big/ \sum_l s_l$ 1 , where s_i is the node strength described as $s_i = \sum_j w_{ij}$.

(ii) Weight dynamics. The weight of each new add edge is initially set to a given value w_0 which is often set

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to 1 for simplicity. But the adding of edge connecting to node i will result in increasing the weight of the other edges linked to node i which is proportional to the edge weights. If the total increase is δ (we will focus on the simplest form: $\delta_i=\delta$), we can get

$$w_{ij} = w_{ij} + \Delta w_{ij} = w_{ij} + \delta \frac{w_{ij}}{s_i} . \quad (1)$$

This will yield the strength increase of node i as:

$$s_i = s_i + \delta + w_0 . \quad (2)$$

The degree distribution of BBV network $P(k) \propto k^{-\gamma_k}$ and the strength distribution $P(s) \propto s^{-\gamma_s}$ yield scale-free properties with the same exponent [11]:

$$\gamma_k = \gamma_s = \frac{4\delta + 3}{2\delta + 1} . \quad (3)$$

The traffic model can be described as follows:

1) All nodes can create packets with addresses of destination, receive packets from other nodes, and forward the packets to their destinations.

2) At each time step, R packets are created with randomly chosen sources and destinations. Once a packet is created, it is placed at the end of the queue if the node has packets waiting to be forwarded.

3) At the same time, the first C_i packets at the head of the queue of each node are forwarded one step to their destinations. If a node has less than C_i packets in its queue, all packets in the queue will be forwarded one step.

4) Upon reaching its destination, the packet is removed from the network.

In our model, three node delivery capability schemes are discussed: (i) each node has the same packet delivery capability (we set C_i=1 for simplicity); (ii) the node delivery capacity is considered to be proportional to the node degree k_i ($C_i=k_i/<k>$); (iii) the node delivery capacity is considered to be proportional to the node strength s_i ($C_i=s_i/<s>$). The total node delivery capability of the whole network is equal to the node number n in these three schemes.

Denote $P_{i \to j}$ as the path between node i and j, which pass through the nodes sequence $x_0(=i), x_1, x_2, \cdots, x_{n-1}, x_n(=j)$, we define

$$F(P_{i \to j}, \alpha) = \sum_{i=1}^{n} x_i^{\alpha} . \quad (4)$$

In our routing strategy, we specify the routing path between i and j as the one makes $F(P_{i \to j}, \alpha)$ minimum under a given tuneable parameter α.

We discussed the situation where x_i is the weight of the edge in the path w_{ij} (WEI stands for this strategy) [9]. In this paper, we explore the other two cases: x_i is the degree of node i ($x_i=d_i$, DEG stands for this strategy), and x_i is the strength of node i ($x_i=s_i$, STR stands for this strategy).

The transfer capacity of networks is most commonly measured by a critical generating rate R_c in those routing strategies [7-9]. A continuous phase transition from free-flow state to congested state occurs at the critical value R_c. In the former state, the numbers of created and delivered packets are balanced, leading to a steady state. The number of accumulated packets increases with time due to the limited delivery capacity or finite queue length of each node in the latter state. We are interested in the critical value R_c which can best reflect the maximum transfer capacity of a network handling its traffic.

The betweenness b_i is often introduced to estimate the possible packet passing through a node i under the given routing strategy, which is defined as

$$b_i = \sum_{s,t} \frac{\sigma(s,i,t)}{\sigma(s,t)} , \quad (5)$$

where $\sigma(s,i,t)$ is the number of paths under the given routing strategy between nodes s and t that pass through node i and $\sigma(s,t)$ is the total number of paths under the given routing strategy between nodes s and t and the sum is over all pairs s, t of all distinct nodes.

A created packet will pass through the node i with the probability $b_i / \sum_{j=1}^{n} b_j$. Thus, the average number of packets that the node i receives at each time step is $Rb_i/(n(n-1))$. Congestion occurs when the number of incoming packets is larger than the outgoing packets, that is $Rb_i / (n(n-1)) \geq C_i$. So the critical packet generating rate R_c is:

$$R_c = \min(C_i n(n-1) / b_i) . \quad (6)$$

3 Simulations and analysis

To get the optimal value of tuneable parameter α, we obtain the critical packet generating rate R_c versus different parameter α in a BBV network with n=100, δ=5, m=5 and ω_0=1 with three different routing strategies. The result of the WEI routing strategy is shown in Figure 1, the DEG routing strategy and the STR routing strategy are presented in Figure 2 and 3 correspondingly.

(For every network, 10 instances are generated and for each instance, we run 10 simulations. The results are the average over all the simulations.)

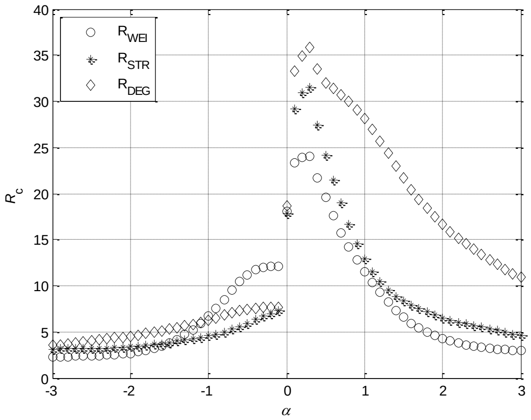


FIGURE 1 R_c VS α. BBV network with n=100, δ=5, m=5, ω_0=1, C_i=1

Figure 1 exhibits the relationship of the critical packet generating rate and the tuneable parameter where each node has the same packet delivery capability. In all three routing strategies, the critical packet generating rate R_c varies with the tuneable parameter α and R_c reaches the peak when α is 0.3. In this case, when α is 0, it is the same as the traditional shortest path routing strategy [7]. Three routing strategies have almost the same critical packet generating rate at α=0. The maximum transfer capacity of the DEG routing strategy (α=0.3) is 92.2% greater than the traditional shortest path routing strategy (α=0) while the STR routing strategy and the WEI routing strategy is 76.7% and 33.3% correspondingly. And the maximum transfer capacity of the DEG routing strategy is better than the other two strategies (13.9 % higher than STR and 49.1% higher than WEI). The figure demonstrates that when each node has the same packet delivery capability, the network transfer capacity reaches the maximum when packets are forwarded through the path whose sum of the 0.3 power of node degree is the minimum.

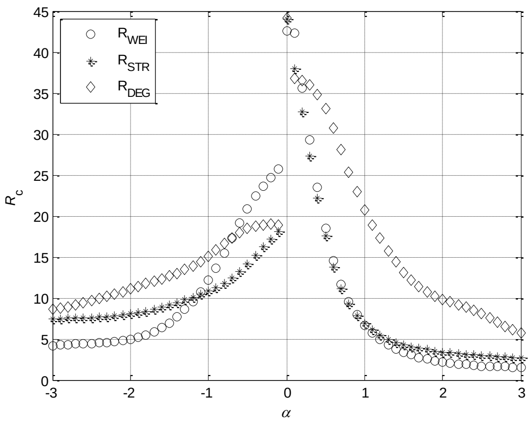


FIGURE 2 R_c VS α. BBV network with n=100, δ=5, m=5, ω_0=1, $C_i=k_i/\langle k\rangle$

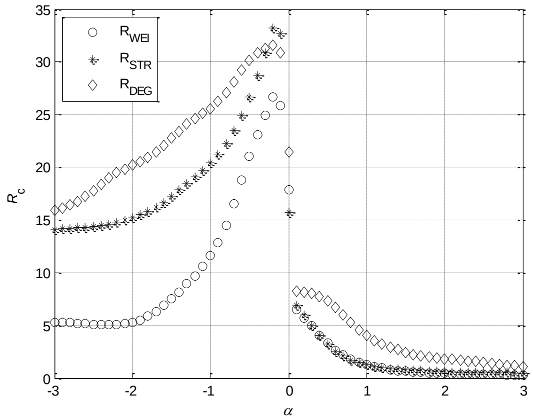


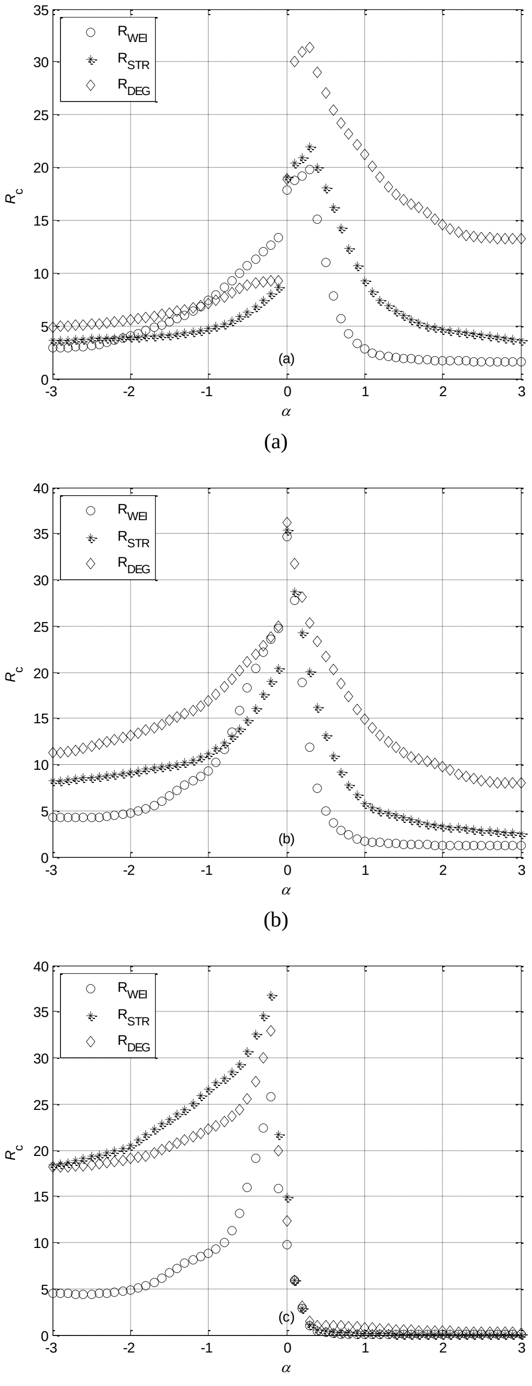
FIGURE 3 R_c VS α. BBV network with n=100, δ=5, m=5, ω_0=1, $C_i=s_i/\langle s\rangle$

The critical packet generating rate versus the tuneable parameter where the node delivery capacity is considered to be proportional to the node degree or the node strength is presented in Figures 2 and 3. The critical packet generating rate R_c reaches the peak when α is 0 while $C_i=k_i/\langle k\rangle$ and α is -0.2 while $C_i=s_i/\langle s\rangle$. And while $C_i=k_i/\langle k\rangle$, all three routing strategies get almost the same maximum transfer capacity at α= 0. The STR routing strategy works better than the other two routing strategies at α=-0.2 when $C_i=s_i/\langle s\rangle$.

To discuss the implication of the weight on critical packet generating rate, we set δ=50 to get the corresponding simulation results in Figure 4.

Comparing Figures 4(a–c) with Figures 1–3, we can obtain that the critical packet generating rate R_c also reaches the peak when α is 0.3, 0, and -0.2 according to different the node delivery capacity. (We also test the situation where δ is 0.5 to find that the simulation results are followed similar patterns.)

As stated above, in those computer generated BBV networks, the critical packet generating rate R_c varies with the tuneable parameter α in all three routing strategies. In the scheme that each node has the same packet delivery capability, the network achieve maximum transfer capacity with the DEG routing strategy when the tuneable parameter α is 0.3. In the scheme that the node delivery capacity is considered to be proportional to the node degree, the network achieve maximum transfer capacity with all three routing strategies when the tuneable parameter α is 0. And in the scheme that the node delivery capacity is considered to be proportional to the node strength, the network achieve maximum transfer capacity with the STR routing strategy when the tuneable parameter α is -0.2.



(a)

(b)

(c)

FIGURE 4 R_c VS α. BBV network with n=100, δ=50, m=5, ω_0=1 (a) C_i=1 (b) $C_i=k_i/<k>$ (c) $C_i=s_i/<s>$

Then we choose the scientific collaboration network [13], which has a giant component of 5835 nodes to test our routing strategy on real world network. Simulation results are shown in Table 1.

As Table 1 shown, the critical packet generating rates reach the peak with the same tuneable parameter as we proposed. It means that our routing strategy is also effective in real world network.

TABLE 1 the critical packet generating rates of the scientific collaboration network

	C_i=1			$C_i=k_i/<k>$			$C_i=s_i/<s>$		
	WEI	DEG	STR	WEI	DEG	STR	WEI	DEG	STR
α=-0.2	2.36	2.10	2.10	5.00	4.99	5.01	7.34	10.06	10.06
α=0	2.47	2.47	2.47	7.15	7.14	7.16	7.01	6.99	7.01
α=0.3	2.49	2.87	2.83	4.15	3.98	3.99	5.81	5.58	5.59

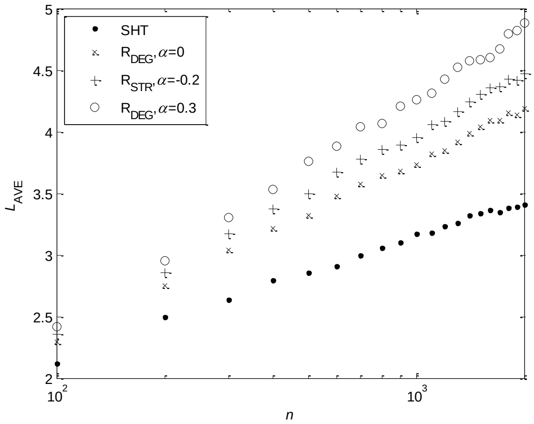


FIGURE 5 L_{AVE} VS n. BBV network with δ=4, m=4 and ω_0=1

The average weighted average length [14] L_{AVE} versus the node number n is reported in Figure 5. Although the weighted average length of DEG scheme and the STR scheme are higher than that of the traditional shortest path [7], the small-world phenomenon, i.e. $L_{AVE} \propto \ln n$, is still maintained. The transfer capacity of weighted network is considerably enhanced at the cost of increasing the average weighted average length. Such a sacrifice may be worthwhile when a system requires large transfer capacity.

4 Conclusion

This paper has proposed routing strategies to enhance the transfer capacity of the BBV weighted networks. The transfer capacity varies with the tuneable parameter and reaches the peak at different tuneable parameter according to three different kinds of node delivery capacity. In the scheme that each node has the same packet delivery capability, the DEG routing strategy achieve maximum transfer capacity when the tuneable parameter α is 0.3. When the node delivery capacity is considered to be proportional to the node degree, all three routing strategies achieve maximum transfer capacity when α is 0. And when the node delivery capacity is considered to be proportional to the node strength, the STR routing strategy achieve maximum transfer capacity when α is -0.2. Meanwhile, the small-world phenomenon of the average weighted average length is still maintained. At last, the test on the scientific collaboration network proves that our routing strategies work well in real world network.

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References

[1] Watts D J, Strogatz S H 1998 *Nature* **393**(6684) 440-2
[2] Barabási A L, Albert R 1999 *Science* **286**(5439) 509-12
[3] Onnela J P, Saramäki J, Hyvönen J, Szabó G, Lazer D, Kaski K, Kertész J, Barabási A L 2007 *PNAS* **104**(18) 7332-6
[4] Newman M E J 2001 *Phys. Rev. E* **64**(1) 016132
[5] Barrat A, Barthelemy M, Pastor-Satorras R, Vespignani A 2004 *PNAS* **101**(11) 3747-52
[6] Pastor-Satorras R and Vespignani A 2007 Evolution and structure of the Internet: A statistical physics approach *Cambridge:* Cambridge University Press
[7] Zhou T 2008 *Physica A: Statistical Mechanics and its Applications* **387**(12) 3025-32
[8] Yan G, Zhou T, Hu B, Fu Z Q, Wang B H 2006 *Phys. Rev. E* **73**(4) 046108
[9] Shao F 2013 *The Scientific World Journal* 378083
[10] Yook S H, Jeong H, Barabási A L, Tu Y 2001 *Phys. Rev. Lett*. **86**(25) 5835-8
[11] Barrat A, Barthélemy M, Vespignani A 2004 *Phys. Rev. Lett.* **92**(22) 228701
[12] Wang W X, Wang B H, Hu B, Yan G, Ou Q 2005 *Phys. Rev. Lett.* **94**(18) 188702
[13] Newman M E J 2001 *PNAS* **98**(2) 404-9
[14] Opsahl T, Agneessens F, Skvoretz J 2010 *Social Networks* **32**(3) 245-51

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