Travel route choice model based on regret theory

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Abstract

Travel route choice behaviour research is a hot issue in the field of urban traffic planning, and it mainly researches the traveller's route choice decisions under uncertainty conditions, which theory includes such as expected utility theory, prospect theory, and regret theory. Based on the analysis of expected utility theory and prospect theory's applicable condition and the insufficiency, this paper establishes a travel route choice model according to regret theory. Study shows that people always try to avoid occur that other options is better than that selected option, and the properties of selected option cannot be replaced each other, which fits regret minimization of regret theory. The travel route choice model based on regret theory is simpler than others, and it is suitable for describing traveller's route choice behaviour under uncertainty conditions.

Keywords: urban traffic, travel route choice, regret theory, Bayesian updating

1 Introduction

Travel route choice is a major decision for a traveller in the process of travelling or before travel. Research on travel route choice behaviour is a hot issue in the fields of urban traffic planning and navigation. Because of traffic network's complexity and time-varying characteristic, and traveller's own differences, this makes traveller's route choice behaviour is uncertain in some degree.

At present, the travel route choice behaviour research mainly references expected utility theory and prospect theory. Expected utility theory has been widely applied in traffic value comparison, and it assumes that people are perfectly rational. Travellers will choose the option which has the largest expected utility according to the complete information those travellers mastered. However, it is difficult to fully master the accurate traffic information for the travellers in fact, and the travellers' preferences and attitudes are not entirely rational, the practical behaviour of travellers' route choice doesn't fully respect the axiomatic system of expected utility theory [1, 2].

Considering travellers' limited rationality as "economic man", Kahneman and Tversky in 1979 raised the prospect theory on the basis of Simon's limited rationality theory, and improved the theoretical model in 1992 [3,4]. Compared with the utility function in expected utility theory, prospect theory introduced the concept of value function; there is a reference point concept for the value function. Reference point is the demarcation point to distinguish the gains and losses. When the reference point uses a different index, the same thing may produce a different result. Wang Yan and Zhang Li proved that under the uncertainty conditions of road network "prospect theory" is more appropriate to describe travellers' decision-making behaviour [5]. Zhang Yang's empirical research also shows the behaviour that people choose vehicles' travel time and paths are consistent with prospect theory under uncertain environment [6]. Luo applied the prospect theory to travel route choice and proved its effectiveness through a example [7].

Prospect theory describes a two stages decision-making model using a two dimensions model of evaluation: valuing outcomes function and weighting of probabilities function. In addition, it involves reference point's definition and using, to distinguish gain or loss. Although the reference point in economics domain is usually unique, in the traffic fields single reference point is not sufficient to solve practical problems. Jou and Kitamura assumes two reference points when they studied travel route choice problem, including the earliest acceptable arrival time and work start time [8]. Schwanen and Ettema set three reference points in the study of route choice behaviour of parents to transfer their children, including the possible departure time, the probability of arriving on time, and late penalties [9]. De Moraes Ramos researches the diversity of reference point in prospect theory [10]. It restricted the application of prospect theory in the travel route choice fields because of the reference point's diversity and complexity.

For a normal traveller, the positive effect is a factor of travel route choice, but the negative consequences that may occur also must be accepted. The researchers tried to looking for a more realistic theory to explain and describe the travellers' route choice behaviour. Among them, Loomes and Sudgen in 1982 [11], Bell in 1982 [12] independently proposed a "regret theory", and they pointed out that the single factor's utility function cannot explain the behaviour of non-rational decision well. People will compare the actual situation and possible

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situation according to their decision-making. If they find their choosing can get better results than other options, they will rejoice. On the contrary, they will feel regret. On the basis of regret theory Casper proposed a random regret minimization model(RRMM), and applied it to travel route choice [13, 14]. RRMM supposed that the satisfaction degree of a travel route not only depends on the utility of selected travel route, but also on the regret of other options' possible better utility. Giselle, etc. compared the expected utility theory, prospect theory and regret theory in travel route choice behaviour prediction, and pointed out that regret theory can be more truly to describe the route choice behaviour than the expected utility theory, and has a simpler form than prospect theory algorithms [15]. Regret theory has only one parameter, namely a regret aversion parameter. When simulating travel route choice behaviour we only need to determine the regret aversion parameter and it is easy to identify according to the foregone experience.

This paper builds a traveller route choice model based on regret theory, and using Bayesian theory to update traveller's regret utility, analyses the travellers' route choice behaviour under uncertain conditions. Then we give a simple numerical example based on a three-link network. At last, we studies the characteristic of regret theory in travel route choice problem.

2 Model based on regret theory

2.1 REGRET UTILITY FUNCTION

Regret theory thinks that travellers' decisions depend not only on the selected route's expected utility, but also on the unselected route's expected utility. If the decision maker finds other unselected routes can produce better result he will feel regret. Conversely, if the selected route's expected result is superior to those unselected routes he will feel delighted [11]. Regret theory is the alternative methods to study expected utility of risk and uncertain selection, and it is mainly used to compare the options' results and options' attributes in practice.

Here we suppose there are two routes for traveller's choosing, as shown in Figure 1. Moreover, travel time is the only evaluation index. Travel times on both routes are uncertain. For travellers they cannot master the exact value of the both routes, but they can know the status' probability p_s of both routes and the travel times in different status (t_s^1, t_s^2).

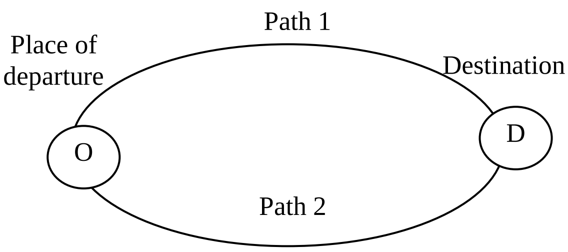

FIGURE 1 Schematic of travel route choice

In expected utility theory, the travellers will choose the route with the largest expected utility (EU). For one route, we can calculate its expected utility:

$$EU = \sum_s \left[p_s \bullet U(t_s) \right], \tag{1}$$

where $U(t_s)$ is a utility function, and it can be represented in multiple forms. Generally, we can use time and cost as index, using linear function or Exponential function. Following two equation is the frequently-used form.

$$U(t_s) = [1 - \exp(\theta \bullet t_s)] / \theta, \tag{2}$$

$$U(t_s) = \alpha \bullet t_s + \beta \bullet c_s + \varepsilon, \tag{3}$$

where θ is a risk aversion parameter, and α, β are the factors of travel time t_s, travel cost c_s. ε is the dimensionless parameter.

Compared with expected utility theory, regret theory supposes that traveller would feel regret or joyful when he finished choosing. The anticipated feeling can be introduced to the utility function as follow.

$$RU(t_s^1) = U(t_s^1) + \varphi[U(t_s^1) - U(t_s^2)], \tag{4}$$

$$RU(t_s^2) = U(t_s^2) + \varphi[U(t_s^2) - U(t_s^1)], \tag{5}$$

where $RU(*)$ is a regret utility function. When $R(0) = 0$, its mean that traveller will not feel regret or joyful. So according to the performance of regret theory, Chorus [14] described it as follows:

$$RU(*) = 1 - \exp(-\lambda \bullet [*]), \tag{6}$$

where $\lambda \in [0, +\infty)$, and it is regret aversion parameter, which reflect the importance of the variable [*]. When λ increases the regret becomes more and more important than joy, and when λ approaches zero regret utility function becomes expected utility function.

Figure 2 is the relation between time or other [*] index and regret utility with parameter λ. Here λ1<λ2<λ3.

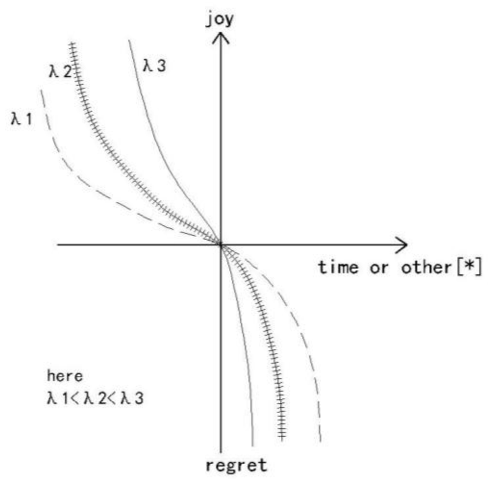

FIGURE 2 The relation between time or other [*] index and regret utility with parameter λ

Regret theory assumes that travellers may have expected utility about each option in regret and joy, and they can add the regret or joy together, and then produce each option's expected regret. In a given status, expected regret is the function of selected option's properties.

$$ERU_1 = \sum_s [p_s \bullet RU(t_s^1)], \tag{7}$$

$$ERU_2 = \sum_s [p_s \bullet RU(t_s^2)]. \tag{8}$$

Travellers will always have an optimum desired arrival time when they go to work from home to companies every day, and too early or to late arriving will suffer some loss. Before travelling them will prediction travel time of each path based on previous travel experience. Here we suppose that the traveller's perception travel time of the two paths T_i^P obey normal distribution, describes as:

$$T_i^P \sim f_i^P(t_i) = N(\hat{t}_i^P, \sigma_i^{P^2}), \tag{9}$$

where, $i = 1,2$. $\hat{t}_i^P$ is the perceived travel time average value of path Y_i, and $\sigma_i^{P^2}$ is the perceived variance.

We suppose the two paths are independent and their travel times are also independent. Traveller will choose path according to minimum regret decision strategy, namely they will choose the path, which has the lowest regret to avoid regret emotions. Chorus gave a regret computational formula of travel route Y_1, as follow:

$$ERU(Y_1) = \int_{-}^{+}\int_{-}^{+} r_1 \bullet [RU(t_2) - RU(t_1)] \bullet f_1^P(t_1) \bullet f_2^P(t_2) dt_1 dt_2 . \tag{10}$$

Here, $RU(t_i)$ is the utility of travel route Y_i. $f_i^P(t_i)$ is the probability density function of perception of travel route Y_i's time. r_1 is the determining factor, $r_1 = 1$ when $RU(t_2) > RU(t_1)$, otherwise $r_1 = 0$.

Similarly, we can calculate the regret $ERU(Y_2)$ of route Y_2.

Assuming travellers select the path, which has the smallest regret finally, and then the selected path's regret is shown in Equation (11).

$$ERU = \min_{i=1,2}(ERU(Y_i)). \tag{11}$$

2.2 BAYESIAN UPDATING FUNCTION

Before travelling traveller will compare the alternative options according to foregoing travel experience and current traffic information gained, and then choose the best travel route. After travelling travellers will evaluate and modify the regret value according to the actual result of the selected option, and form the updated travel experience for the next travel at the same conditions. Because of the uncertainty of choice condition and the limitation of the traveller's knowledge, they cannot estimate the travel time very accurately. Here we suppose that the perceived travel time is continuous random variable and obeys the normal distribution, after received the newest traffic or travel information the travellers will update the perceived travel time distribution of each alternative option, and we can use Bayesian updating method to simulate the updating process or result.

After obtaining the newest traffic, information travellers will update expected travel time distribution for each route combined with previous travel experience. Because traffic information is time varying, there is a deviation between traveller's perceivable travel time T_i^I and actual travel time T_i. Therefore, we suppose that perceivable travel time obeys normal distribution, namely $T_i^I \sim f_i^I(t_i)$. Standard deviation σ_i^I is the perception about travel time of travellers. According to Bayesian updating theory, the updating travel time T_i^u also obeys normal distribution.

$$T_i^u \sim f_i^u(t_i) = N(\hat{t}_i^u, \sigma_i^{u^2}). \tag{12}$$

Then we can obtain Equation (13) by Bayesian updating inference [16].

$$f_i^u(t_i) = f(t_i^P \mid t_i^I) = \frac{f(t_i^I \mid t_i^P) \bullet f_i^P(t_i)}{f_i^I(t_i)}, \tag{13}$$

$$f_i^I(t_i) = \int_{-}^{+} f(t_i^I \mid t_i^P) \bullet f(t_i^P) dt_i^P . \tag{14}$$

T_i^u's mean value and variance are Equation (15) and Equation (16) respectively.

$$\hat{t}_i^u = \frac{(1/\sigma_i^P)^2 \bullet \hat{t}_i^P + (1/\sigma_i^I)^2 \bullet t_i^I}{(1/\sigma_i^P)^2 + (1/\sigma_i^I)^2}, \tag{15}$$

$$\sigma_i^u = \sqrt{\frac{\sigma_i^{P^2} \bullet \sigma_i^{I^2}}{\sigma_i^{P^2} + \sigma_i^{I^2}}}. \tag{16}$$

After updating the distribution of the expected travel time, travellers will make decisions again. Now the regret of travel route choice Y_1 can be calculate by Equation (17).

$$ERU^I(Y_1) = \int_{-}^{+}\int_{-}^{+} r_1 \bullet [RU(t_2) - RU(t_1)] \bullet f_1^u(t_1) \bullet f_2^u(t_2) dt_1 dt_2 . \tag{17}$$

In a similar way, we can calculate the regret of Y_2.

Travellers still select the route which has the smallest regret. At the same time considering the probability of received information, the regret of the desired route which traveller selected can be gained through Equation (18).

$$ERU^{I}=\int_{-}^{+}\int_{-}^{+}[\min_{i=1,2}(ERU^{I}(Y_i))]\bullet f_1^{I}(t_1)\bullet f_2^{I}(t_2)dt_1dt_2 \;. \quad (18)$$

2.3 MODEL BASED ON REGRET THEORY

After travellers determined all routes' regret, they will carry out route choice according to regret. There are two methods of travellers' route choice.

Firstly, they will choose the route, which has the smallest regret directly without considering the impact of the route factors, and this is named as deterministic selection model.

$$P(r,s,n)=\min\{R_i,R_j,R_k\}. \quad (19)$$

Secondly, we can establish discrete choice model with considering the impact of the route factors. The probability of each route can be calculated by Logit model as Equation (20).

$$P(r,s,n)=\exp[-\sigma t(n)/\bar{t}]/\sum_{i=1}^{m}\exp[-\sigma t(i)/\bar{t}], \quad (20)$$

where, $P(r,s,n)$ is the assignment probability which OD T(r,s) in route n. $t(n)$ refers the regret value of the route n, and $\bar{t}$ is the average regret value of all routes. σ is the assignment parameter. m is the number of valid travel routes.

3 Examples

Here we reference the example in the literature 17 and replace part of the data. There are three routes i, j, k between the place of departure and destination for traveller n. So he or she has three travel routes or options, including alt_i, alt_j and alt_k. The property of travel routes includes travel time x (min) and travel cost y (yuan). Here we suppose that the time and the cost are fixed for every route. And $alt_i=\{x_i=75,y_i=1\}$, $alt_j=\{x_j=45,y_j=3\}$, $alt_k=\{x_k=30,y_i=20\}$.

The regret of one route or option equals to the compare between it and other two options, namely:

$$R_i=\max\{R_{ij},R_{ik}\},$$
$$R_j=\max\{R_{ji},R_{jk}\}, \quad (21)$$
$$R_k=\max\{R_{ki},R_{kj}\}.$$

Here the regret value of one route is the sum of the inter-comparison of two options' every property. For example, we can define the binomial regret as follow:

$$R_{ij}=\varphi_x(x_i,x_j)+\varphi_y(y_i,y_j). \quad (22)$$

And $\varphi_x(x_i,x_j)$, $\varphi_y(y_i,y_j)$ are the regret function of property. They can be described as follow:

$$\varphi_x(x_i,x_j)=\max\{0,\beta_x(x_j-x_i)\},$$
$$\varphi_y(y_i,y_j)=\max\{0,\beta_y(y_j-y_i)\}. \quad (23)$$

Here β is the parameter of property regret, and it reflects the relative importance between different properties. Refer to actual perceived value, we suppose that the β of travel time is -1.0/min, and the β of travel cost is -0.5/yuan.

Equation (22) and Equation (23) reflect that the regret function of property is the linear function about property difference.

And we can calculate the regret compared of alt_i, alt_j.

$$R_{ij}=\max\{0,-1.0\times(45-75)\}+\max\{0,-0.5\times(3-1)\}=30. \quad (24)$$

The regret of traveller is travel time, and not travel cost.

In a similar way, we can gain:

$R_{ik}=\{0,-1.0\times(30-75)\}+\{0,-0.5\times(20-1)\}=45$,

$R_{ji}=1$, $R_{jk}=15$, $R_{ki}=9.5$, $R_{kj}=8.5$.

Thus, it is concluded that: $R_i=45$, $R_j=15$, $R_k=9.5$. With regret theory, model traveller preference structure is $alt_k>alt_j>alt_i$, travel will choose alt_k. This reflects that travel time is the main influence factor. However, in fact some person will choose route according travel cost when they have enough time. Moreover, it is different to expected theory that it cannot be totally compensation or replace between properties. When certain property beyond the ability of traveller's expense, he or she will not choose the option.

Under the framework of utility maximization, the poor performance of the travel time can be replaced by the good performance. Nevertheless, in regret theory framework, people prefer to choose the option without regret in all the properties.

Now we use Equation (20) to analysis the probability of every route according to Logit distribution. Here σ=3.5, and

$P(i)=\exp(-3.5\times45/23.16)/[\exp(-3.5\times45/23.16)$
$+\exp(-3.5\times15/23.16)+\exp(-3.5\times9.5/23.16)]$
$=0.003$

$P(j)=0.302$, and $P(k)=0.695$.

In order to further study of regret features and applications, we suppose that the time of three routes is varying every day and they are independent of each other. Each has two status, including normal and congestion.

The normal travel times of alt_i, alt_j and alt_k respectively are 55, 40 and 30 min. The congestion travel times respectively are 75, 55 and 60 min. For every route the probability of congestion is the 0.50. Therefore, we can gain 8 status having equal probability. The travel costs of each route respectively are 1, 3 and 20. Here we do not update regret. Table 1 shows the properties of each route in different status.

TABLE 1 Properties of each route in different status

Status	alt_i		alt_j		alt_k	
	x	*y*	*x*	*y*	*x*	*y*
1	55	1	40	3	30	20
2	55	1	40	3	60	20
3	55	1	55	3	30	20
4	55	1	55	3	60	20
5	75	1	40	3	30	20
6	75	1	40	3	60	20
7	75	1	55	3	30	20
8	75	1	55	3	60	20

Table 2 shows the regret value of each route in different status.

TABLE 2 Regret value of each route in different status

status	$R(alt_i)$	$R(alt_j)$	$R(alt_k)$	choose
1	25	10	9.5	alt_k
2	15	1	28.5	alt_j
3	25	25	9.5	alt_k
4	0	1	14.5	alt_i
5	45	10	9.5	alt_k
6	35	1	28.5	alt_j
7	45	25	9.5	alt_k
8	20	1	13.5	alt_j
average	26.25	9.25	15.375	alt_j

From table 1 and table 2, we can know that under different status conditions travellers will choose different route based on regret theory. Totally under 8 status travellers choose alt_i 1 times, alt_j 3 times, and alt_k 4 times. However, using average regret value, travellers' choosing order is $alt_j > alt_k > alt_i$. Travellers would like to alt_j. From the comparison of the properties of the routes, we can know route alt_j has a relative balanced index in travel time and travel cost.

If we use expected utility function to analysis the travel route choice, we can gain the result with little significant difference. Based on travel time traveller will choose alt_k, however based on travel cost they would like to choose alt_i. In addition, if the two index has different weight, travellers would have varied choose, among them some may be similar to model based on regret theory.

In travel route choice domain based on regret theory, it is the regret relative to the best options playing an important role in the decision-making, not joy. This shows the character of properties of options in choosing process, namely bad utility of option's property cannot be directly compensated by another good utility of option's property. Travel route choice rules are to choose the option with the smallest regret. When the regret value is greater than a certain threshold, its value is beyond of the ability of traveller's ability to pay, and this will lead to cannot make decisions and they will delay choose through looking for more information available.

Now we use Equation (20) to analysis the probability of every route according to Logit distribution. Here σ =3.5, and

$$P(i) = \exp(-3.5\times 26.25/16.96)/[\exp(-3.5\times 26.25/16.96) + \exp(-3.5\times 9.25/16.96) + \exp(-3.5\times 15.38/16.96)] = 0.023$$

$P(j)=0.762$, and $P(k)=0.215$.

Using regret theory or expected utility theory to measure the relative attraction of options for selection result is different, under the framework of discrete choice within econometrics, attract degree's difference between alternative options means that the predict choice probability will be different.

4 Conclusions

In this paper, firstly compared expected utility theory, prospect theory and regret theory in travel route choice study, and then a model based on regret theory and Bayesian updating method is established. The conclusion shows that when travellers choose travel routes they will try to avoid those unselected routes are better than the selected route. At the same time they think one worse property of route cannot be replaced by another better property. All of these are corresponding with regret minimization of regret theory. In addition, travel route choice model based on regret theory is relatively simple.

When each property of one option is the same or better than the other option's each property, the regret minimization model with generic selecting aggregation and multiple attribute decision making can be simplified as the utility maximization model.

Since the travel route choice problem involves not only the establishment of the utility function, it also involves the travellers' information updates mechanism and the travellers' choice characteristics; we should carry further study through actual travel route choice data.

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