Solving electrically-large objects RCS based on 3-D vector parabolic equation method

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Abstract

The vector parabolic equation (VPE) method is introduced to calculate bistatic RCS of three-dimensional (3-D) electrically-large objects and polarization effects are fully taken into account. According to an approximate form of the vector wave equation and divergence-free condition the VPE was derived in this paper. The numerical results conducted on the scattering from perfectly conducting cube show the VPE agree with the exact method, and computation time is acceptable compared with the traditional full wave method.

Keywords: vector parabolic equation, electrically-large objects, radar cross section

1 Introduction

The Parabolic Equation (PE) is an approximation of wave equation, and it first was introduced by Leontovich and Fock [1, 2] in 1940s to treat the problem of diffraction of radio waves around the Earth. Recently, it have been applied to radar cross section (RCS) calculations [3, 4]. The scalar PE method expands the pseudo-differential operator of the scalar wave equation, discretizes objects with a series of planar element, the solution is marched in that direction from one transverse plane to the next, thus reducing the full three-dimensional problem to a sequence of two-dimensional calculations, which enhance the efficiency of computing greatly. We can get RCS at all scattering angles by near-field/far-field transformations and rotating the paraxial direction. The PE techniques may bridging the gaps between rigorous numerical methods [5] and asymptotic methods [6]. Using the PE method one can avoid both the limits of CPU time and memory by the rigorous numerical methods and those of narrow applications by the asymptotic methods, especially for electrically-large objects scattering problem.

In order to treat polarization effects fully for electromagnetic scattering, the Vector PE is obtained by coupling component scalar parabolic equations via suitable boundary conditions on scatterers. In this paper the vector parabolic equation method was introduced to calculate bistatic RCS of 3-D electrically-large objects.

2 3-D Parabolic equation

2.1 3-D SCALAR PARABOLIC EQUATION

In this paper, the time dependence of the fields is assumed as exp(-jωt). We work in Cartesian coordinates (x, y, z) and start with 3-D scalar wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+k^2n^2\psi=0\,, \tag{1}$$

where k is the wave number, and n is refractive index. Choosing the positive x-direction as the paraxial direction and defining the reduced field u by

$$u(x,y,z)=\exp(-ikx)\psi(x,y,z)\,. \tag{2}$$

The scalar wave equation in terms of u is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+2ik\frac{\partial u}{\partial x}+k^2(n^2-1)u=0 \tag{3}$$

and it can be formally factored as

$$\left\{\frac{\partial}{\partial x}+ik(1-\sqrt{Q})\right\}\left\{\frac{\partial}{\partial x}+ik(1+\sqrt{Q})\right\}u=0 \tag{4}$$

in which the pseudo-differential operator Q is given as

$$Q=\frac{1}{k^2}\frac{\partial^2}{\partial y^2}+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}+n^2\,. \tag{5}$$

Equation (4) can be reduced to

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$$\frac{\partial u}{\partial x} = -ik\left(1-\sqrt{Q}\right)u\,, \tag{6}$$

$$\frac{\partial u}{\partial x} = -ik\left(1+\sqrt{Q}\right)u\,. \tag{7}$$

Equation (6) is the called forward parabolic equation, which is most importance in many cases. Using first-order Taylor expansions of the square root $\sqrt{Q}$:

$$\sqrt{Q} \approx 1+\frac{Q-1}{2}\,. \tag{8}$$

The 3-D scalar Parabolic Equation is given by

$$\frac{\partial u}{\partial x} = \frac{ik}{2}\left[\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+n^2-1\right]u\,. \tag{9}$$

2.2 3-D VECTOR PARABOLIC EQUATION

Outside the objects the electric and magnetic fields $\vec{E}$ and $\vec{H}$ satisfy the vector wave equation

$$\begin{cases}\nabla^2\vec{E}+k^2\vec{E}=0\\ \nabla^2\vec{H}+k^2\vec{H}=0\end{cases}. \tag{10}$$

In this paper we solve in terms of electric field $\vec{E}$ and magnetic field $\vec{H}$ that can be obtained from $\nabla\times\vec{E} = -j\omega\mu_0\vec{H}$, if required. We write

$$\vec{E} = \vec{E}^i + \vec{E}^s\,, \tag{11}$$

where $\vec{E}$, $\vec{E}^i$ and $\vec{E}^s$ are total, incident and scattered fields, respectively. The three fields all satisfy the vector wave Equation. The reduced scattered filed *u* in *x*-direction is

$$u^s = e^{-jkx}\vec{E}^s\,.$$

As scalar parabolic equation approximation, the vector wave equation can be factored three scalar parabolic equations for the components (u_x^s, u_y^s, u_z^s) of u^s:

$$\begin{cases}\dfrac{\partial u^s{}_x}{\partial x} = \dfrac{ik}{2}\left[\dfrac{1}{k^2}\left(\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}\right)+n^2-1\right]u^s{}_x\\ \dfrac{\partial u^s{}_y}{\partial x} = \dfrac{ik}{2}\left[\dfrac{1}{k^2}\left(\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}\right)+n^2-1\right]u^s{}_y\\ \dfrac{\partial u^s{}_z}{\partial x} = \dfrac{ik}{2}\left[\dfrac{1}{k^2}\left(\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}\right)+n^2-1\right]u^s{}_z\end{cases}. \tag{12}$$

In what follows, we consider perfectly conducting objects embedded in a vacuum (*n*=1), so the tangential electric field must be zero on the surface of scattering objects. This gives the following system of equation:

$$\begin{cases}n_z u_y^s(P) - n_y u_z^s(P) = -e^{-jkx}(n_z E_y^i(p) - n_y E_z^i(p))\\ n_x u_z^s(P) - n_z u_x^s(P) = -e^{-jkx}(n_x E_z^i(p) - n_z E_x^i(p))\\ n_y u_x^s(P) - n_x u_y^s(P) = -e^{-jkx}(n_y E_x^i(p) - n_x E_y^i(p))\end{cases}, \tag{13}$$

where *P* is a point on the surface of scattering objects and (n_x, n_y, n_z) is the outer normal to the surface at *P*.

In order to obtain a well-determined system we must introduce the divergence-free condition of Maxwell's equation because the three equations of in (13) are not independent. The divergence-free equation is:

$$\frac{\partial\vec{E}_x^s}{\partial x}+\frac{\partial\vec{E}_y^s}{\partial y}+\frac{\partial\vec{E}_z^s}{\partial z}=0\,. \tag{14}$$

Combining the first Equation of (12) and (14), we can get the vector parabolic equation as

$$\frac{i}{2k}\left(\frac{\partial^2 u_x^s}{\partial y^2}+\frac{\partial^2 u_x^s}{\partial z^2}\right)+iku_x^s+\frac{\partial u_y^s}{\partial y}+\frac{\partial u_z^s}{\partial z}=0\,. \tag{15}$$

3 Implementation aspects

3.1 FINITE-DIFFERENCE SCHEME

As scalar parabolic equation method [7], the vector parabolic equation can be solved using finite-difference scheme. To 3-D object scattering problem, we use a double-pass method [8], where the field is first propagated assuming the object is not present at the next range. For this first pass the equations are separable and the scheme can be factored into tridiagonal matrices, which can be inverted efficiently with Gauss pivot methods. In the second pass the field is recalculated taking the object into account, using the first pass results as boundary values for a small transverse region enclosing the scatterer. A sparse matrix formulation implementing the electromagnetic boundary conditions is used for this second pass, ensuring that polarization effects are fully taken into account pass the equations are separable.

3.2 DOMAIN TRUNCATION

For domain truncation in the transverse, we added Perfectly Matched Layer (PML) as the truncation boundary conditions. We construct 3-D PML by replacing coordinate *y* and *z* with complex coordinate $\hat{y}$ and $\hat{z}$ given by

$$\hat{y} = y - i\int_0^y \sigma(\zeta)d\zeta\,, \tag{16}$$

$$\hat{z} = z - i\int_0^z \sigma(\xi)d\xi\,, \tag{17}$$

where

$$\begin{cases}\sigma(\zeta)=0, & y_a \le y \le y_b \\ \sigma(\zeta)>0, & \text{other}\end{cases} \tag{18}$$

$$\begin{cases}\sigma(\xi)=0, & z_a \le z \le z_b \\ \sigma(\xi)>0, & \text{other}\end{cases}. \tag{19}$$

Integration domain with PML absorbing boundary condition is shown in Figure 1. It is composed by the area of $[y_a, y_b]\times[z_a, z_b]$.

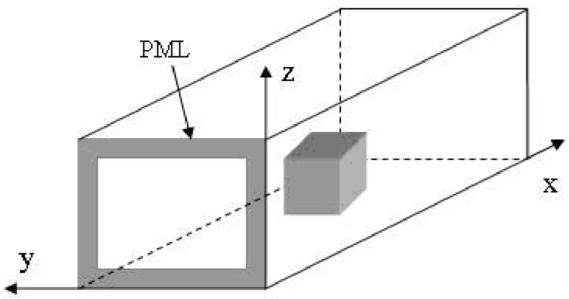

FIGURE 1 Integration domain with PML absorbing boundary condition

3.3 FAR-FIELD FORMULAS

When receiver polarization has to be taken into account, the total bistatic RCS is defined as

$$\sigma_{\vec{t}}(\theta,\varphi)=\lim_{r\to\infty}4\pi r^2\frac{\left|\vec{E}^s(x,y,z)\cdot\vec{t}\right|^2}{\left|\vec{E}^i(x,y,z)\right|^2}, \tag{20}$$

where $x=r\cos\theta,\ y=r\sin\theta\cos\varphi,\ z=r\sin\theta\sin\varphi$.

We assume the receiver is polarized along vector $\vec{t}$. If the incident field is a plane wave of unit amplitude, we can get the equation:

$$\sigma_{\vec{t}}(\theta,\varphi)=$$

$$\frac{k^2\cos^2\theta}{\pi}\left|\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\vec{E}^s(x_{0,}y,z)\cdot\vec{t}e^{-ik\sin\theta(y\cos\varphi+z\sin\varphi)}dydz\right|^2 \tag{21}$$

in which x_0 can usually be chosen as 10λ (λ is the wavelength of incident wave). The total RCS is obtained by summing *x*, *y*, *z* components of RCS.

4 Numerical results

In this numerical example the incident field is a plane wave with its amplitude set to be 1.0, and incident angle is 0°. The incident source with its wavelength of 1m (frequency 0.3 GHz) is vertically polarized and propagating in a vacuum (*n*=1), which illuminated a perfectly conducting cube 12λ on each side. We use PML absorbing boundary conditions at computation domain. To obtain the bistatic RCS (0°-180°), we need six rotated PE runs for VPE. Figure 2 and Figure 3 show bistatic RCS of perfect conducting cube 12λ on each side for E and H plane patterns respectively. The results proved the validity of the present method. Using the VPE method, the CPU consumed by this example was under 10 min on desktop computer, but this case would be quite stressful for traditional Method of moments [8].

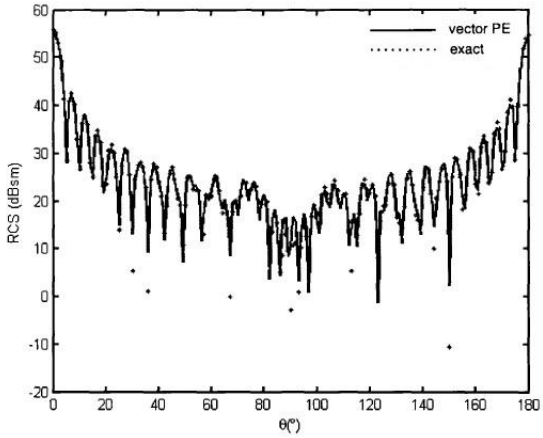

FIGURE 2 Bistatic RCS of perfect conducting cube 12λ on each side (E plane)

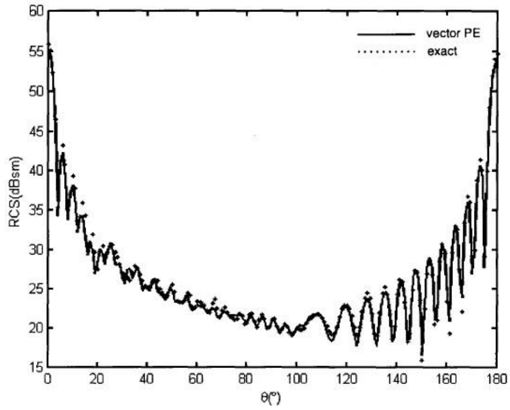

FIGURE 3 Bistatic RCS of perfect conduct cube 12λ on each side (H plane)

5 Conclusions

The vector parabolic equation allows accurate treatment of polarization effects within the paraxial constrains. The combination of VPE formulation with the rotated PE methods provides a powerful tool for electromagnetic scattering problem. The results of example proved the efficiency of the VPE method to calculate Electrically-large Objects RCS. The work presented here is limited to perfectly conducting objects embedded in a homogeneous background.

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