Application of PID-type iterative learning control for DC motor

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Abstract

Iterative learning control is a new control technology, which is a branch of intelligent control theory and particularly suitable for the controlled object with repetitive motion. In this paper, a PID-type iterative learning control for DC motor based on the characteristics of repeating motion of DC motor was proposed and the convergence of iterative learning control algorithm was analysed. The input of controlled system in current cycle was amended by the error achieved between the system output and the desired trajectory in previous iteration. It was tested that PID-type ILC had good performance and stability through a large number of simulations and the experiments of the velocity tracking are done by MATLAB software. The results showed that the velocity tracking precision of DC motor was higher and the error was smaller with the increasing number of iterations. The velocity tracking error was close to zero. It was also shown that the motor could fully track the given desired trajectory in some certain iteration. It was also revealed from simulation results that the proposed control strategy was valid and effective for the DC motor.

Keywords: PID control, DC motor, PID-type iterative learning control

1 Introduction

With the feature of simple structure, small size, light weight, reliable operation, easy maintenance, good servo performance, fast response speed and good stability, the DC motor is widely applied in the fields of servo system, factory automation and defence industry [1]. In recent years, with the rapid developing of electronic technology and micro-controller technology, it has a much higher demand to the velocity control precision of DC motor when it executes certain task repetitively. Owing to the difficulty of DC motor in setting up accurate mathematical model, the selection of control method seems utmost important in order to allow the system to own higher control precision, stability and higher tracking performance.

In 1984, Arimoto et al. first introduced the initial explicit formulation of iterative learning control [2, 3]. Since the ILC algorithm was proposed, a very large number of approaches have been considered. Its control actually imitates human learning character, achieves tracking task in limited time of simple in principle and needs higher control performance. The input of controlled system in current cycle is amended by the error achieved between the system output and the desired trajectory in previous iteration. ILC is a technique to control the systems with a defined task repetitively in a limited and constant time interval. Ref [4] proposed the robust optimal design problem and analyzed the convergence of iterative learning control. A novel model-based method was presented so that ILC based on a quadratic performance criterion was revisited [5]. Ref [6] designed the ILC algorithms based on parametric optimization approach. ILC is widely concerned because of no precise mathematical model [7-9]. Lee applied ILC technique to improve tracking control performance in batch processes [10]. Ref [11] offered a 2-Dimensional systems theory based ILC. A driven data constrained norm-optimal iterative learning control framework was proposed [12]. Generally speaking, iterative learning control system can realize precise control characteristic by self-regulation disvelocity without the precise mathematical model. Therefore, it promotes a great development of D-type iterative learning control [13], P-type iterative learning control [14, 15], PD-type iterative learning control [16], PI-type iterative learning control [17], PID-type iterative learning control theories [18-24]. In recent years, iterative learning control theory is rapid developed both at home and abroad, and there are many achievements.

With the continual advances of control theory, the PID controller is still the most commonly used controller in the process control industry [25]. Traditional PID control can’t track the real-time velocity timely and precisely, and its control is poorer. Liu Haishan directly applied PID control for the control of brushless DC motor. This method has strong robustness and poor traceability [26]. Jia Hongping applied sliding mode variable structure control for the control of brushless DC motor, and it has strong robustness, good traceability and buffeting [27].

This paper applied PID-type iterative learning control algorithm for the control of DC motor based on repetitive operation DC motor system. The convergence of the algorithm is analysed in theory and simulated for the mathematical model of DC motor in MATLAB. The results show that the simulation result agrees with the

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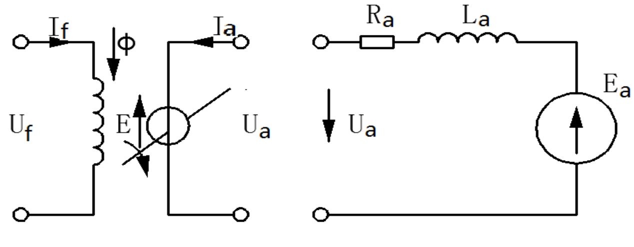
theoretical analysis. With the increasing of iterations, the accuracy is higher and higher, the error is smaller and smaller which is close to zero. In addition, the motor can fully track the given desired trajectory in some certain iterations.

The paper is organized as follows. The mathematical model of the DC motor is established in the Section 2. In Section 3, the PID-Type iterative learning control algorithm is discussed and designed. In Section 4, we make numerical simulations make for the DC motor transmission system using the proposed control method. Conclusions are drawn in Section 5.

2 Mathematical Model

In modern industry, DC motor has been the execution terminal extensively used in servo system. This paper regarded brushless electromagnetic DC motor as research object and the mathematical model is set up for it [28]. Figure 1 is the working principle and equivalent circuit diagram of DC motor.



(a) (b)

FIGURE 1 Diagram of working principle and the equivalent circuit of DC motor ((a) working principle, (b) equivalent circuit)

Armature circuit voltage balance equation can be obtained:

$$U_a = E_a + R_a I_a = C_E \Phi_n + I_a R_a = K_E n + R_a I_a \,. \quad (1)$$

Equation (2) is dynamic equation of DC motor:

$$u_a = K_E n + R_a i_a + L_a \frac{di_a}{dt}\,, \quad (2)$$

where R_a is the loop resistance. I_a is loop current. E_a is loop induced electromotive force. U_a is the terminal voltage of circuit. C_E is the constant of electromotive force. n is the motor speed. K_E is the electromotive force generated by unit speed. Φ_n is the flux.

Equation (3) is electrodynamics balance equation:

$$T_e = B\omega + T_L + J\frac{d\omega}{dt} = C_E \Phi i_a = K_T i_a\,, \quad (3)$$

where T_e is instant electromagnetic torque. T_L is load torque. B is damping coefficient. J is rotational inertia. K_T is a constant of torque. ω is angel velocity.

Supposing the initial conditions of the motor is zero, and the load of the motor is constant. The transfer function can be obtained after the LAPLACE conversion:

$$G(s) = \frac{\omega(s)}{U(s)} = \frac{K_T}{L_a J s^2 + (R_a J + L_a B)s + R_a B + K_T K_E}\,. \quad (4)$$

3 PID-Type Iterative Learning Control Algorithm

Iterative learning control is mainly to learn in repetition, and improve certain control objective through iteration control. The algorithm is simple, and it can realize the actual motion track of the unknown object in the given time horizon. The algorithm does not rely on the mathematical model of the controlled plant.

The PID-type iterative learning control law of linear time-varying system was given [2]:

$$u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_{k+1}(t) + \Phi e_{k+1}(t) + \Psi \int_0^t e_{k+1}(\tau)d\tau\,. \quad (5)$$

The form of singular linear continuous system is:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}. \quad (6)$$

In the formula, Γ , Φ , Ψ are gain matrixes. k is iterations. A, B, C are real matrixes.

If the systems, which are described in Equations (5) and 6 satisfy the following conditions:

(1) $\|I - CB\Gamma\| \le \bar{\rho} < 1$;

(2) Supposing the initial condition of each iteration is the same, namely, $x_k(0) = x_0$, $(k = 1, 2, 3, \cdots)$, $y_0(0) = y_d(0)$. There is $y_k(t) = y_D(t)$, $\forall t \in [0, T]$ when $k \to \infty$.

Proof:

In accordance with reference [13], it can be inferred from Equation (6) and condition (2): $y_{k+1}(0) = Cx_{k+1}(0) = Cx_k(0) = y_k(0)$.

Therefore, $e_k(0) = 0$, the output error of the time of k+1 is:

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - \int_0^t C\varphi(t,\tau)B(\tau)[\Gamma(\tau)\dot{e}_k(\tau) \\ &+ L(\tau)e_k(\tau) + \Psi(\tau)\int_0^\tau e_k(\delta)]d\tau \end{aligned}. \quad (7)$$

Supposing $G(t,\tau) = C(t)B(\tau)\Gamma(\tau)$, then

$$\int_0^t G(t,\tau)\dot{e}_k(\tau)d\tau = G(t,\tau)e_k(\tau)|_0^t - \int_0^t \frac{\partial}{\partial_\tau}G(t,\tau)e_k(\tau)d\tau = CB(t)\Gamma(t)e_k(t) - \int_0^t \frac{\partial}{\partial_\tau}G(t,\tau)e_k(\tau)d\tau \quad (8)$$

Substitute (8) into (7), then

$$e_{k+1}(t) = [I - CB(t)\Gamma(t)]e_k(t) + \int_0^t \frac{\partial}{\partial_\tau}G(t,\tau)e_k(\tau)d\tau - \int_0^t C\varphi(t,\tau)B(\tau)L(\tau)e_k(\tau)d\tau - \int_0^t\int_0^\tau C\varphi(t,\tau)B(\tau)\Psi(\tau)e_k(\sigma)d\sigma d\tau \quad (9)$$

Make bound norm of both ends of (9), then

$$\begin{aligned}\left\|e_{k+1}(t)\right\| &\le \|I - C(t)B(t)\Gamma(t)\|.\left\|e_k(t)\right\| + \int_0^t \left\|\frac{\partial}{\partial_\tau}G(t,\tau)\right\|\left\|e_k(\tau)\right\|d\tau + \int_0^t \|C\varphi(t,\tau)B(\tau)L(\tau)\|\left\|e_k(\tau)\right\|d\tau + \int_0^t\int_0^\tau \|C\varphi(t,\tau)B(\tau)\Psi(\tau)\|\left\|e_k(\sigma)\right\|d\sigma d\tau \le \\ &\|I - C(t)B(t)\Gamma(t)\|.\left\|e_k(t)\right\| + \int_0^t b_1\left\|e_k(\tau)\right\|d\tau + \int_0^t\int_0^\tau b_2\left\|e_k(\sigma)\right\|d\sigma d\tau\end{aligned} \quad (10)$$

where $b_1 = \max\left\{\begin{array}{l}\sup\limits_{t,\tau\in[0,T]}\left\|\frac{\partial}{\partial\tau}G(t,\tau)\right\|, \\ \sup\limits_{t,\tau\in[0,T]}\|C(t)\Phi(t,\tau)B(\tau)L(\tau)\|\end{array}\right\}$, $b_2 = \sup\limits_{t,\tau\in[0,T]}\|C(t)\Phi(t,\tau)B(\tau)\Psi(\tau)\|$.

The $e^{-\lambda t}$ is multiplied at both ends of Equation (10), and $\lambda > 0$, then

$$\left\|e_{k+1}\right\|_\lambda \le \tilde{\rho}\left\|e_k\right\|_\lambda, \quad (11)$$

where $\tilde{\rho} = \bar{\rho} + b_1\frac{1-e^{-\lambda T}}{\lambda} + b_2\left(\frac{1-e^{-\lambda T}}{\lambda}\right)^2$.

It can be inferred from condition (1) that there is $\tilde{\rho} < 1$ if λ is biggest.

Therefore $\lim\limits_{k\to\infty}\left\|e_k\right\|_\lambda = 0$.

The proof is ended.

This paper makes the output signal sequence $u(t)$ converge uniformly optimal signal $u^*(t)$ based on PID-type iterative learning control theory. In addition, the output $y_k(t)$ conforms with the expectation trajectory tracking of $y_D(t)$ in $t \in [0,T]$. The control process of PID-type iterative learning control algorithm is shown in Figure 2. The idea of ILC is to gradually revise imperfect control input using the error between system output and the desired trajectory and realize perfect tracking in a finite time interval.

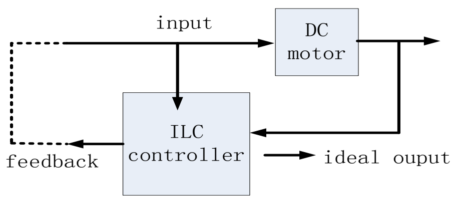


FIGURE 2 Iterative learning control

4 Simulation

To confirm the algorithm's effectiveness, the parameters of DC motor are shown in Table 1. According to the selected parameter in Formula 4, the motor’s transfer function can be obtained as Equation 12.

$$G(s) = \frac{100}{s^2 + 10s + 1} \quad (12)$$

Discrete the transfer function and sampling time is 1ms. The linear system of controlled object is obtained as Equation (13).

$$\begin{cases} X(k+1) = AX(k) + Bu(k) \\ y_V(k) = CX(k) \end{cases}, \quad (13)$$

where $A = \begin{bmatrix} 1 & 0.001 \\ 0 & 0.9753 \end{bmatrix}$, $B = \begin{bmatrix} 0.0001 \\ 0.1314 \end{bmatrix}$ and $C = [1 \quad 0]$.

The specific simulation parameter is $\Gamma = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.6 \end{bmatrix}$, $\Phi = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\Psi = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$.

The ideal trajectory is the $y_d(t) = \sin(3t)$. The simulation results are velocity tracking and error respectively in different iterations, and the results are as shown in Figure 3-10. The iterations are 5, 10, 20 and 30. From the results, the velocity tracking differs from the ideal track in the first iteration cycle. With the increase of iterations, the velocity tracking trajectory approaches to the ideal trajectory more and more. Meanwhile the velocity tracking error is becoming smaller and smaller.

Especially after the iterations reach twenty times, the velocity tracking trajectory nearly coincides with the ideal track. Based on the above, it can be confirmed that PID-type iterative learning control algorithm is very suitable for the velocity trailing of DC motor with repeating motion property. In addition, the more the times of repetition is, the more the velocity trailing accuracy is, and the smaller the velocity trailing error is. It can be deduced that the motor can fully track the given expectation trajectory in certain value of iterations. Meanwhile, the control performance is verified of PID-type iterative learning control algorithm is verified by the experiments.

TABLE 1 Relevant parameters of DC motor

parameter	numerical value
K_T	0.001
K_E	0.01
R_a	0.1 Ω
L_a	10mH
J	0.001kg.m2
B	0

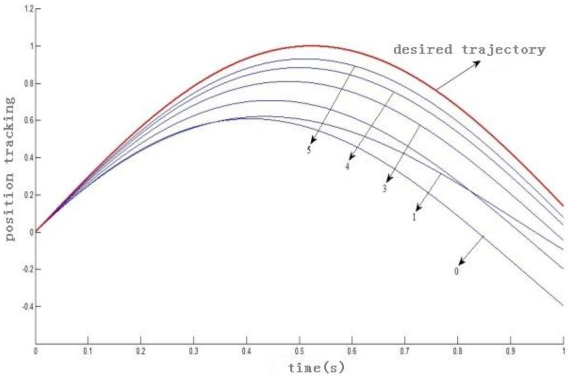


FIGURE 3 Velocity tracking at the iterations=5

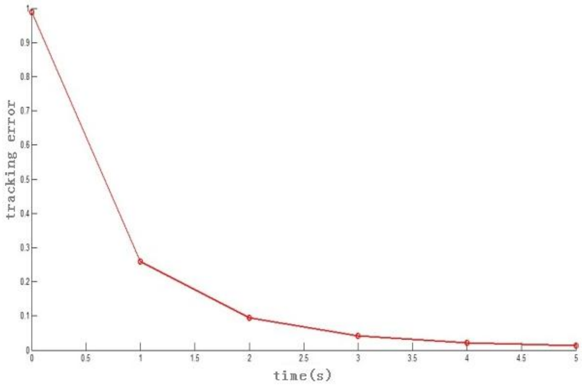


FIGURE 4 Tracking error at the iterations=5

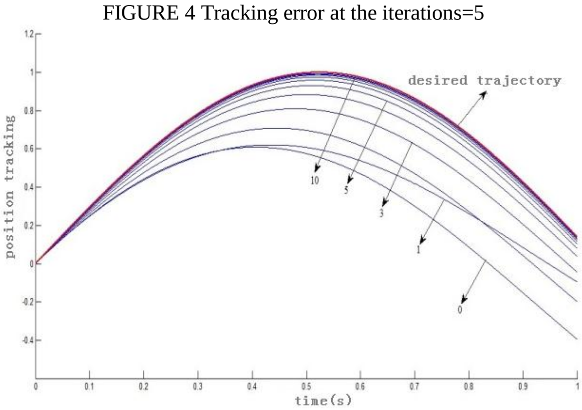


FIGURE 5 Velocity tracking at the iterations=10

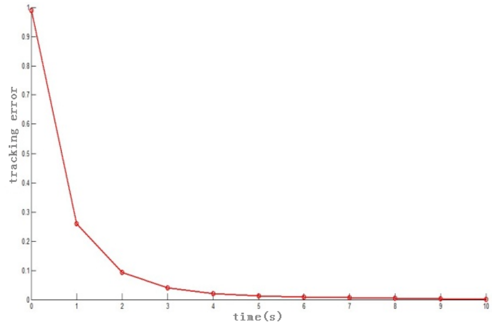


FIGURE 6 Tracking error at the iterations=10

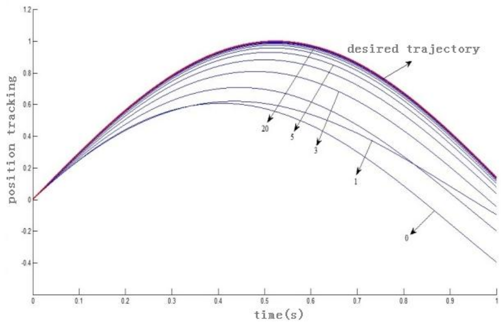


FIGURE 7 Velocity tracking at the iterations=20

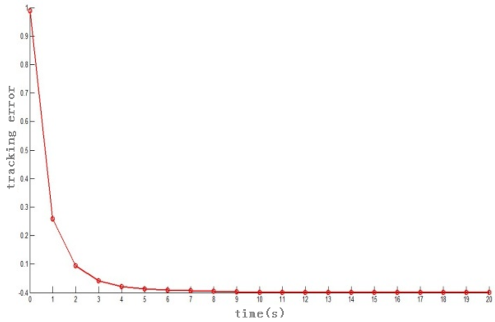


FIGURE 8 Tracking error at the iterations=20

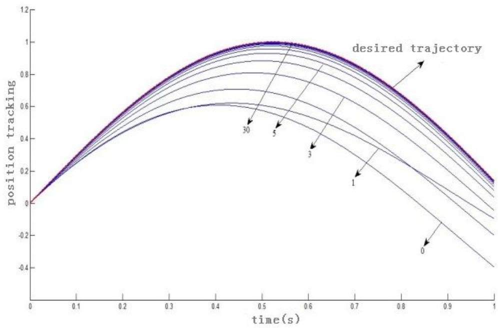


FIGURE 9 Velocity tracking at the iterations=30

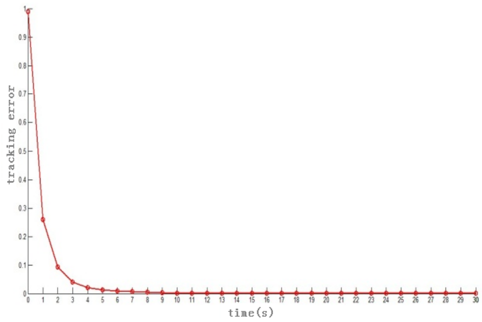


FIGURE 10 Tracking error at the iterations=30

5 Conclusion

The paper applied PID-type iterative learning control algorithm to control the velocity of DC motor with the feature of repeating motion. By the simulation results we can obtain the following conclusions:

(1) With the increase in the number of repetitive motion of the DC motor, the velocity tracking errors by a PID-type iterative learning control algorithm is getting smaller and smaller, according to the simulation shows that when the iterative learning is more than 20 times, the actual velocity and the ideal one substantially coincident.

(2) The motor can fully track the given expectation trajectory in certain value of iterations. At the same time, it is proved that iterative learning control algorithm can ceaselessly evaluate the control performance of the system by taking full advantage of the periodicity or repeatability of the dynamic behaviour. It can amend the control signal of the system, and improve the control performance of the system. It is also revealed from simulation results that the proposed control strategy is valid and effective for the DC motor.

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