Research on the calculation method of the penetration probability of projectile fragments based on Monte Carlo

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Abstract

This paper studies the simulate calculation about the coordinates of burst based on Monte Carlo method and the projectile fragments scattered point coordinate .According to scattered point coordinates in fried and the projectile fragmentation, this paper puts forward the concept of penetration coefficient; and through calculating the penetration coefficient, it comes a conclusion that the bigger the coefficient becomes, the probability of target damage is higher. It calculates the probability of a single fragment penetrating target and the overall target damage by using the calculation method of probability distribution. The dynamic simulation of debris' kinetic penetration and the simulation of the changes of energy and the damaged probability are carried out. In this way, the results of the simulation are analyzed, which verifies the correctness of the theory.

Keywords: Monte Carlo method, the kinetic energy projectile fragments, Transmission Probability, damage probability, Penetration simulation.

1 Introduction

Modern war is the information war under nuclear deterrence. In this war style, information and firepower are the two pillars. Information leads reconnaissance, surveillance, communications and command and control in modern warfare [1,2]. Firepower is closely related to weapons, which ultimate purpose is to destroy the target. So we can say, target damaging is one of the most important links of the weapon usage in information warfare [3]. Hitting the core protection is also an important factor to decide the outcome of a war. Therefore, it is very important to strengthen the research on target damage field. One of the important bases for the research of target damage is the formation of debris cloud after the target exploding (i.e. the damage field) that contributions to hit target destructive effect [4, 5]. So a lot of researchers at home and abroad on target damage analysis focus on the sample multiple fragments. Calculation of the number of fragments involves in the encounter conditions such as the target being hit, debris characteristics, and target characteristics and so on. These finally reflect on two problems of debris dynamic distributing density and debris on the target area [6]. These various methods are only in the theory of computation that the exact data of the space debris cannot be effectively obtained at the moment of target exploding, which greatly restricts the study of dynamic simulation. Therefore, this paper simulate calculates the projectile burst coordinates space, debris scattered point coordinates through the Monte Carlo method, and judges the degree of target's damage by the projectile fragment penetration coefficient. Finally, this paper turns on the damage probability of the whole target board.

2 The simulate calculation of projectile and projectile fragmentation parameters based on Monte Carlo method

2.1 COORDINATE SIMULATION OF THE BURST POINT BASED ON MONTE CARLO METHOD

It is efficient and feasible to be free from simplified measures that the analysis method cannot be cancelled by using the Monte Carlo method [7,8] to calculate the coordinates of the point. In the ballistic end, exploding projectiles are randomly distributed in the flight direction, its distribution law is called the burst point distribution law. Take the geodetic coordinates as the observation point; the target can be regarded as a point. To establish x-y plane coordinate system on the target plane, and take the goal as the origin, assume the projectile point distribution law accords with Gauss normal distribution, it can be generally speaking that the coordinate point of fried is regarded as independent random variables, and all that about the burst point spread centre obey the Gauss normal distribution. If each explosion point coordinate is (x_i, y_i), burst point spread centre is $(\bar{x}, \bar{y})$, and fried difference point distributed in x and y direction can be neglected, then $\sigma_x = \sigma_y = \sigma a$ .The definition of circular error probability can describe the projectile point accuracy, thus we can get CEP radius: $R_{0.5} = 1.1774\sigma$.

Without considering systematic errors, the burst point of projectile projection in the target plane (x', y') obeys to two-dimensional normal distribution $\sigma_x = \sigma_y = R_{0.5} / 1.774$,

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then from random sampling formula:

$$\begin{aligned} x' &= x + (\alpha \cdot R_{0.5}/1.1774), \\ y' &= y + (\beta \cdot R_{0.5}/1.1774). \end{aligned} \quad (1)$$

In these, independent u_1 and u_2 are uniformly distributed random numbers which obey [0, 1].

2.2 THE SIMULATE CALCULATION OF THE PROJECTILE FRAGMENTATION DISTRIBUTION POINT COORDINATE

Among the number of fragments that are generated by the projectile explosion is *n* uniform distribution. At the moment of explosion, assuming the debris cloud is spherical, the effective damage radius is R according to Monte Carlo method, and the *i-th* fragment's coordinates of the point can be determined by the formula:

$$x_i' = x' + R\xi_1 \cos(2\pi\xi_2)\,, \quad (2)$$

$$y_i' = y' + R\xi_1 \sin(2\pi\xi_2)\,, \quad (3)$$

In these, (ξ_1, ξ_2) is a set of discrete uniform random number.

3 Based on the probability of penetration damage calculation

3.1 PENETRATION CODFFICIENT CALCULATION

After calculating burst point position and the projectile fragments scattered simulation coordinate information, we use damage analysis on target based on burst point of effective damage range and projectile debris effective point [9,10]. Assuming the projectile fragments produced by the explosion are in uniform distribution and the same size; and for fragments, its initial velocity is v_0, quality is m_0. When debris penetrates target board with the perspective of θ, it is as shown in Figure 1.

Due to the time which debris flies to the target is short, speed attenuation could be ignored. Thus the initial kinetic energy of a single projectile fragments is $E_0 = \frac{1}{2} v_0 m_0^2$. The penetration damage on the target is caused by fragments' kinetic energy, and the energy which the target penetrated by fragment can be calculated by the next type:

$$E_p = K_1 \sigma_b \frac{\pi}{4} (2r)^2 \frac{d}{\cos\theta}\,, \quad (4)$$

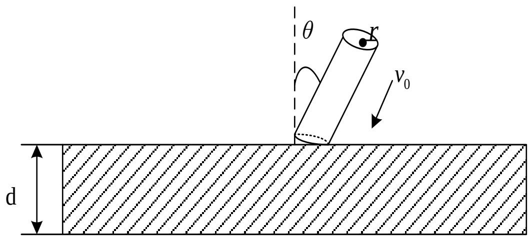

FIGURE 1 The schemes of fragments of projectile penetration target board.

While K_1 is proportional coefficient, and related to target surface material properties and the impact velocity, $K_1 = 0.92 + 1.023v^2 \times 10^{-6}$ if the impact speed is no more than $2500 m/s$; σ_b is critical stress of target materials; d is the thickness of the target board; r is radius of fragment [8].

Assuming δ is penetration coefficient and shows the degree of penetration by the projectile fragments about the target. Then s can be expressed as the ratio between the initial kinetic energy of the projectile fragments and the energy needed from penetrating the objective target plate. So:

$$\delta = \frac{E_0}{E_p} = \left(\frac{1}{2} m_0 v_o^2\right) \Big/ K_1 \sigma_b (2r)^2 \frac{d}{\cos\theta}\,. \quad (5)$$

The shape, size, materials, and the impact angle and velocity of projectile fragments influence on the penetration coefficient. While the penetration coefficient is larger, the probability of the target is mutilated.

3.2 THE PROBABILITY CALCULATION OF TARGET PENETRATION OF SINGLE FRAGMENT

Because the target has metal skin protection, the target area S of stereographic projection S_{Ts} is divided into two parts by the projectile fragments in the process of damage --horizontal plane stereographic projection S_l and vertical plane stereographic projection S_h. The ratio of target projection area could be calculated by expression (6).

$$\begin{cases} \lambda_l = \dfrac{S_l}{S_{Ts}} = \dfrac{S_l}{S_l + S_h} \\ \lambda_h = \dfrac{S_l}{S_{Ts}} = \dfrac{S_h}{S_l + S_h} \end{cases}, \quad (6)$$

and, $\begin{cases} S_l = S \cdot \sin\theta \\ S_h = S \cdot \cos\theta \end{cases}$.

Assuming that random variables Δv_l and Δv_h are respectively the differences between the pieces of the target velocity v_0 and ballistic limit v_{50h}, so:

$$\begin{cases} \Delta v_l = v_{fc} - v_{50l} \\ \Delta v_h = v_{fc} - v_{50h} \end{cases}, \tag{7}$$

If Δv is converted into and standard forms y_{i1} and y_{ih}:

$$\begin{cases} y_{il} = \dfrac{\pi}{8}\dfrac{\Delta v_l}{\sigma_T} = \dfrac{\pi}{8} x_{il} \\ y_{ih} = \dfrac{\pi}{8}\dfrac{\Delta v_h}{\sigma_T} = \dfrac{\pi}{8} x_{ih} \end{cases}, \tag{8}$$

Having used Monte Carlo method to get the spread-coordinates of fragments: x_{il}, x_{ih}, y_{i1}, y_{ih} can be calculated from the spread-coordinates of fragments.

Therefore, the probability of fragments penetrating target can be calculated from $P_h = P_l\lambda_l + P_h\lambda_h$, among them:

$$\mathrm{P}_l = \begin{cases} 0 & y_{il} \le -4 \\ 1 - [0.5 + \dfrac{\sin|y_{il}|}{2}]^{\alpha}, & -4 < y_{il} < 0 \\ [0.5 + \dfrac{\sin|y_{il}|}{2}]^{\alpha} & 0 \le y_{il} < 4 \\ 1 & y_{il} \le -4 \end{cases}, \tag{9 a}$$

$$\mathrm{P}_h = \begin{cases} 0 & y_{ih} \le -4 \\ 1 - [0.5 + \dfrac{\sin|y_{ih}|}{2}]^{\alpha}, & -4 < y_{ih} < 0 \\ [0.5 + \dfrac{\sin|y_{ih}|}{2}]^{\alpha} & 0 \le y_{ih} < 4 \\ 1 & y_{ih} \le -4 \end{cases}, \tag{9 b}$$

$$\alpha = \begin{cases} 1 - 0.612|x_i| + 0.7667(x_i)^2 \\ + 3.44 \times 10^{-3}|x_i|^{0.01}, |x_i| \le 1 \\ 1.66 - [2.6896 - \\ (0.5|x_i| - 1.63)^2]^{0.5}, -4 < y_{il} < 0 \end{cases}. \tag{9 c}$$

3.3 THE PROBABILITY CALCULATION OF GENERAL TARGET DESTRUCTION

In this article, the kinetic energy of the projectiles' fragments produced by the explosion target is mainly used to destroy the target; therefore, the random variables including debris kinetic energy and the spatial distribution of the debris cloud which need to be considered when calculating the probability $P_d(R)$ of target destruction. The probability distribution of a biological explosion is not only point-symmetric, but also continuous and accord with normal distribution form [11, 12]. Due to the symmetry, we only need to calculate a probability distribution of the point of damage to the diameter axis OR to calculate explosion damage probability $P_d(R)$. The axis diameter of OR of damage point is the radius of the axis spreading beyond debris cloud from projectile exploded, and OR is a variable, as shown in Figure 2. In the figure, φ_v is the scattering angle of fragments.

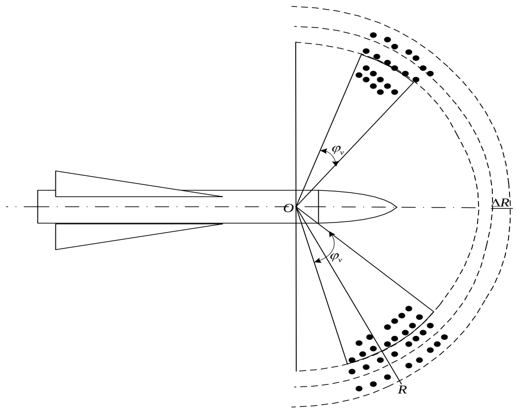

FIGURE 2 The schemes of fragment dispersion

As shown in Figure 2, centring for projectile warheads and divided into concentric ring area on the coordinate plane by the point symmetry properties of damage points [13,14], each ring is continuously distributed on the average radius R of the corresponding probability $P_d(R)$; what's more, the area of each ring is $2\pi R\Delta R$. Because damage probability distribution accords with normal distribution, so:

$$\mathrm{P}_d(R) = 1 - \frac{1}{\sqrt{2\pi\sigma_{ex}}}\int_0^{\infty} \exp[-\frac{(R-\bar{R})^2}{2\sigma_{ex}^2}]dR, \tag{10}$$

Among them, σ_{ex} is square error of R, and $\bar{R}$ is expectation of R.

Thus, we can calculate the integral form about the target damage area A:

$$\begin{aligned} A &= \int_0^{\infty} 2\pi RQP_d(R)dR \\ &= 2\pi\int_0^{\infty} RdR - \int_0^{\infty}\int_0^{R} \frac{\sqrt{2\pi R}}{\sigma_{ex}} \exp[-\frac{(R-\bar{R})^2}{2\sigma_{ex}^2}]dRdR \end{aligned}. \tag{11}$$

4 Simulated analysis

In order to verify the correctness of the theory of penetration, Assume that the quality of the projectile is $6g$, the initial speed of the projectile fragments is $1500m/s$, the thickness of the target is $2cm$, we simulate under this condition and get the following simulation results. Figure 3 shows the acceleration changes of the projectile fragments in the process of penetration, it can be seen that the acceleration before 5 μs is in increasing trend. At this time, the projectile fragments has not to reached the target board, but the projectile fragments starts to penetrate target until penetrate through the target as time goes on. In the process, the acceleration is decreasing. Figure 4 shows the change of projectile fragments velocity vector, it can be clearly seen from the figure that speed is attenuating during the whole process of penetration. Due to ignoring the attenuation of the projectile fragments' quality in the process of penetration and the gravity because of the small pieces of quality, the energy of the fragments, especially the kinetic energy in the process of the penetration is in a state of decay. The theory is verified in Figure 5. Figure 5 is the kinetic energy of the projectile fragments change during the process of penetration; Figure 6 shows the energy changes of the target material during penetration. The decrease of fragments' kinetic energy is bound to lead target material energy increasing according to the law of conservation of energy. Seen from Figure 6, the energy of target material is in increasing trend. As a result, the simulation verifies the correctness of the theory of penetration.

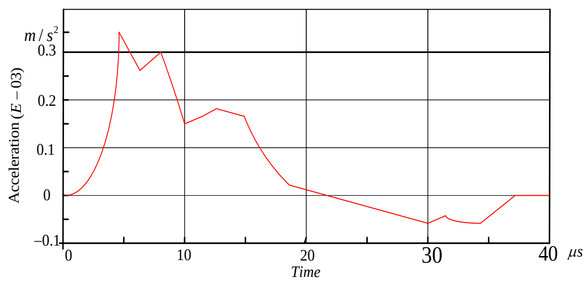

FIGURE 3 Acceleration changes of the projectile fragments in the process of penetration.

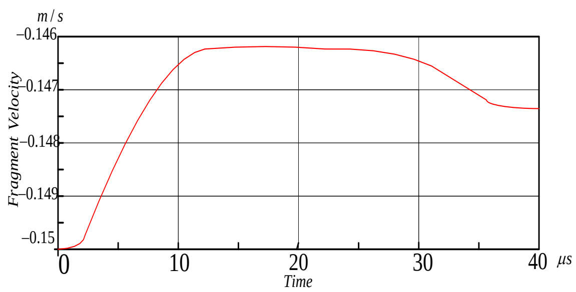

FIGURE 4 Velocity changes of the projectile fragments in the process of penetration

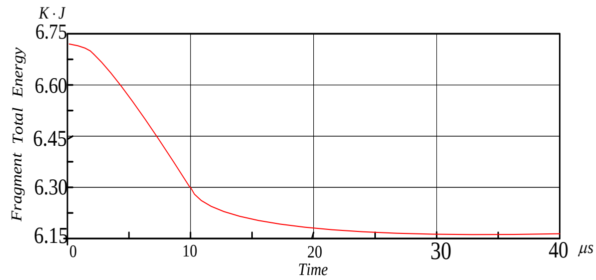

FIGURE 5 Energy changes of the projectile fragments in the process of penetration.

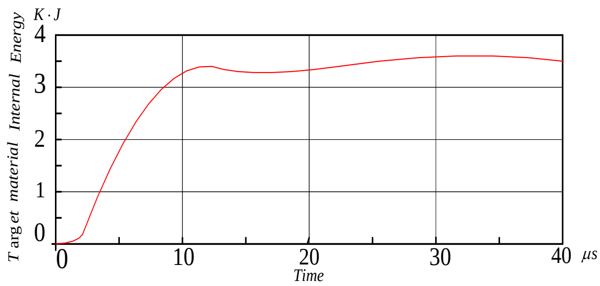

FIGURE 6 Energy changes of the target material in the process of penetration.

In order to analyze the influence of target kill probability by debris penetration more clearly, the influence of the kill probability by debris cloud radius and the initial speed were simulated, the simulation results are obtained as shown in Figure 6 and 7. It can be seen from Figure 6, with the increase of flying debris cloud radius, kill probability decreases. When debris cloud radius is 2m, the kill probability is the largest, when $R > 10m$, the kill probability approaches to zero; at that time, the projectile fragments on the target board has almost with no damage. As can be seen from Figure 7, the kill probability has close relationship with the projectile fragments initial velocity. With the increase of initial velocity, damage probability increases.

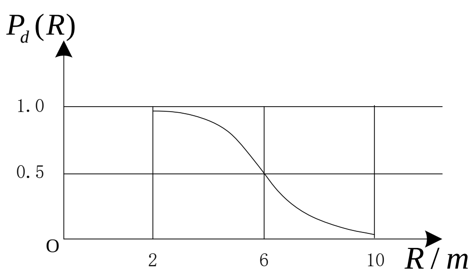

FIGURE 7 The influence of debris cloud radius to kill probability

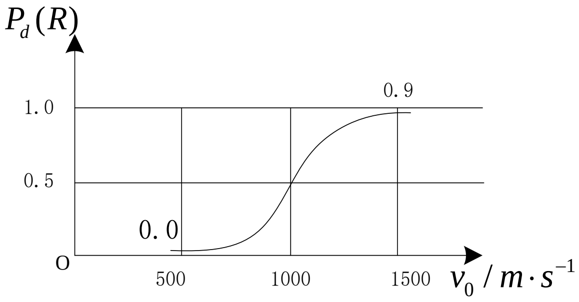

FIGURE 8 The influence of v_0 to kill probability.

5 Conclusions

Through the theoretical calculation of the Monte Carlo method, we obtained simulation coordinate of the burst point and projectile fragments scattered point. Under the condition of effective burst point and effectively projectile fragments scattered point coordinates, the penetration coefficient was calculated, and the conclusion was that the greater penetration coefficient was, the greater the probability of the target was mutilated; For pieces of the projectile penetration mutilate further analysis, we calculated the probability of fragments through the target board and overall target kill probability by using the method of calculating probability distribution, and through software simulation we got the corresponding results: based on the projectile fragments through target board, the fragments of kinetic energy in the reducing process is with the increasing energy of target board; target kill probability decreases with the increase of debris cloud radius, and with the increase of fragment velocity increases. The simulation result demonstrates the validity of the theory further.

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