Multi-state system reliability assessment based on Bayesian networks

Xiaonan Zhang[1, 2*], Xiaoyong Lu[2]

[1]*College of Field Engineering, PLA University of Science and Technology, Nanjing 210007, China*

[2]*The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing 210007, China*

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Abstract

This paper considers a problem of multi-state system reliability modelling and assessment. By using the advantages of uncertainty reasoning and figurative expression of Bayesian network, a new method of modelling and assessment of multi-state system reliability based on BN is proposed to determine the nodes of BN and the multiple states of components of system, and to give the probability of each state and then utilizing conditional probability distributing (CPD)to describe the relationship among the component states, so as to express the states of correlated nodes and build a BN model of multi-state system. The model can clearly express the multiple states of system and component and the state probability, and also call directly calculate the system reliability on the basis of multiple state probabilities of component, thereby carrying out qualitative analysis and quantitative assessment of multi-state system reliability. By means of an example of multi-state radar system, we give the detailed multi-state system reliability analysis process based on BN. This paper not only proves the effectiveness of assessment of multi-state system reliability based on BN, but contributes to good help of complex system reliability, safety analysis.

Keywords: multi-state system, Bayesian networks, reliability assessment

1 Introduction

In traditional binary reliability framework, both systems and components can only take two possible states: completely working and totally failed. However, engineering systems typically have multiple partial failure states in addition to the above-mentioned completely working and totally failed states. Reliability analysis considering multiple possible states is known as multi-states reliability analysis. Multi-state reliability analysis recognizes the multiple possible states of engineering systems, and enables more accurate system reliability analysis.

Traditionally, system reliability has been analysed from a binary perspective assuming the system and its components can be in either of two states: completely functioning or failed. However, many systems that provide basic services, such as telecommunications, gas and oil production, transportation and electric power distribution, operate at various levels of performance as opposed to the binary perspective. These types of systems may provide a service or function at degraded component performance levels. Therefore, it is essential to model and analyse them accordingly. For these systems, multi-state system reliability methods have been proposed as a more appropriate modelling and computational approach.

The idea of multi-state system was first touched as early as in 1968 by Hirsch et al [1]. It was systematically introduced and studied in 1970s by Barlow and Wu [2], EI-Neweihi et al. [3] and Ross [4] by considering a component or a system having more than two possible states. In their work, the primary concepts of multi-state reliability were studied, including system structure function, minimal cut (path) set, relevancy and coherency. The results by the early studies on multi-state reliability were generalized in the work of Griffith [5], Natving [6], Hudson and Kapur [7], and Block and Savits [8]. The early advances in multi-state reliability theory were summarized by EI-Neweihi and Proschan [9].

An important issue is how to model practical system in the multi-state context through careful analysis and definition. Many binary reliability models [10] have been extended to multi-state reliability models, such as the series-parallel system models [2, 11], the k-out-of-n system models [12], the weighted k-out-of-n system model [13], the network system models [14], etc. There might be more than one way to extend a binary reliability model to the multi-state context. For example, in Barlow and Wu's definition of multi-state series-parallel system [2], the state of a parallel subsystem is equal to the state of the best component. However, in Levitin's definition of multi-state series-parallel systems, the capacity of a parallel subsystem is equal to the sum of the capacities of its constituent components. Under traditional definition of multi-state k-out-of-n: G system [3, 15], the system is in state j or above when at least k components are in state j or above. Huang et al. proposed the model of generalized multi-state k-out-of-n: G system by allowing different

* ***Corresponding author*** e-mail: zxn8206@163.com

requirements of the number of components on different states [12, 16]. The model of multi-state consecutive system was also redefined [17]. The binary network reliability models have also been extended to multi-state versions by allowing the links and/or the nodes to have more than two possible states [14, 18, 19].

One way to analyse multi-state systems is using a binary variable to represent a single state of a component [20]. The problem is that there will be dependencies among variables that characterize the same component. The stochastic process approach is a more universal approach in modelling and evaluation of power systems [21]. Because the stochastic process approaches require equation solving whose computation burden can be significantly influenced by the number of components and the number of states, the stochastic approach can only be applied to relatively small systems. Levitin et al. developed the Universal Generating Function (UGF) approach to evaluate multi-state systems [14, 22], which can be used to deal with a wide range of multi-state systems. Like in the reliability evaluation of binary systems, Monte-Carlo simulation can be used for the evaluation of multi-state systems [18]. But compared to analytical algorithms, the main disadvantage of this approach is that it is not computationally efficient, especially for large systems with a large number of components.

With the concerning about the multi-state system reliability by related scholars, the multi-state system reliability theory has been some progress, but these methods there are some limitations, which is at the exploration preliminary stage.

In recent years, Bayesian network (BN) has found applications in, e.g., software reliability [13–16], fault finding systems [17–23], and maintenance modelling [24, 25]. One important feature that makes BN appealing is the possibility of combining different sources of information to provide a global safety assessment. Bouissou et al. [13] report on the experience of a hierarchical construction of a BN to combine different sources of evidence in the reliability analysis of complex software systems. On a similar line, Fenton et al. [14] showed that the robustness and well-founded underlying theory of BN can provide significant advantages. Wooff et al. [15] designed software tests using BN, and concluded that BN are well suited for these problems.

Because BN is good at analysing the uncertainty and correlation of random variables, BN technology applying in system reliability assessment can well make up for existing assessment methods. BN graphical expression function and conditional probability diagram (CPD) can make the relationship expression between systems and components more intuitive and clear. Some researchers constitute BN modelling framework which is particularly easy to use in interaction with domain experts, also in the system reliability field [25-29]. Common aims and goals are currently being recognized by researchers in classical reliability theory and the BN community, and examples of fields of fruitful cooperation include probabilistic inference for fault detection and identification, monitoring, maintenance, and prediction. However, as far as the complex multi- state system reliability modelling and assessment, there is no systematic study and conclusion.

In this paper, by using the advantages of uncertainty reasoning and figurative expression of Bayesian network, a new method of modelling and assessment of multi-state system reliability based on BN is proposed to determine the nodes of BN and the multiple states of elements of system, and to give the probability of each state and then utilizing conditional probability distributing (CPD)to describe the relationship among the element states, so as to express the states of correlated nodes and build a BN model of multi-state system. The model can clearly express the multiple states of system and elements and the state probability, and also call directly calculate the system reliability on the basis of multiple state probabilities of elements, thereby carrying out qualitative analysis and quantitative assessment of multi-state system reliability. Analysis of practical examples proves the effectiveness of assessment of multi-state system reliability by using BN method.

This paper is organized as follows. In the next section, we introduce basic concepts of Bayesian Network theory. In the section 3, we present the two state system reliability modelling based on BN. Section 4 presents multi-state system reliability model. In section 5, by means of an example of multi-state radar system, we show the detailed multi-state system reliability analysis process based on BN. Finally, section 6 concludes the paper.

2 Bayesian Network theory

BN is probabilistic networks based on graph theory. Each node represents a variable and the arcs indicate direct probabilistic relations between the connected nodes. Variables are defined over several states. The BN allow taking into account time by defining different nodes to represent the variables at different time slices.

BN is directed acyclic graphs used to represent uncertain knowledge in Artificial Intelligence [15]. A BN is defined as a couple: G ((N,A),P), where (N,A) represents the graph; N is a set of nodes; A is a set of arcs; P represents the set of probability distributions that are associated to each node. When a node is not a root node, i.e. when it has some parent nodes, the distribution is a conditional probability distribution that quantifies the probabilistic dependency between that node and its parents.

In accordance with the definition of BN conditional probability:

$$P(A/B)=\frac{P(B/A)}{P(B)}, \tag{1}$$

where $P(B)$ is the prior probability, $P(A/B)$ for the posterior probability.

Supposed A is a variable, there are n states $a_1, a_2, ..., a_i, ..., a_n$, according to the total probability equation:

$$P(B) = \sum P(B / A = a_i) P(A = a_i), \quad (2)$$

the posterior probability $P(A/B)$ can be calculated.

With the conditional independence, BN can carry out two-way reasoning, not only forward reasoning, derived from the prior probability to posterior probability, which is from reasons to results, but also derived from the posterior probability to prior probability with the formula, which is from results to reasons.

3 Two state system reliability modelling based on BN

Application BN for systematic assessment in a straight form, we do not have to calculate the system minimal cut sets and minimal path sets, avoiding non-payment computing. In the case, that fault tree (FT) has been established and the FT can be directly mapped into BN.

FT is a kind of analysis method from the whole to part and from the top level to the down level according to varieties of fault reasons. The structure of BN model is correspondence with the FT; the difference is that BN makes the various fault reasons analysis, from the part to the overall, from the down to the top showing a branch shape. The establishment approach of two state systems BN models based on FT are as following:

1) Identify and model the relevant variables and their interpretation. Each basic event of FT corresponds to the root node in BN; each logic gate of FT establishes the corresponding middle node in BN; the same basic events appearing multiple times of FT can be expressed in a root node in BN.

2) Establishment a directed acyclic graph. According to the logic gate and the corresponding BN nodes, the directed arc which links the root with the leaves is expressed as parent and offspring relationship.

3) Giving the conditional probability of each variable, generating Conditional Probability Diagram (CPD). Corresponds to the FT, the priori probability of each root in BN is given. For each logic gate, the additional equivalent of CPD is given for the corresponding node. Based on the logic relationship of each gate, such corresponding CPD can be automatically generated.

Example: a system composed of three valves C_1, C_2, C_3, Figure 1a is the system reliability block diagram. System function is defined as the passage fluid flow from A to B, normal state as a "pass", failure state "broken".

According to the reliability block diagram, FT is established in Figure lb, in which T is expressed as the system failure event (top events), that X_i is expressed as the state of component i, M is an intermediate state of the event. According to the above rules, BN is shown in Figure lc, the root node xi is expressed as the basic event; that the leaf nodes t is expressed as the middle nodes; in CPD, 1 is expressed as fault, 0 is normal.

After the establishment of BN, we apply BN inference algorithm [23], such as the Equation (3) shows that:

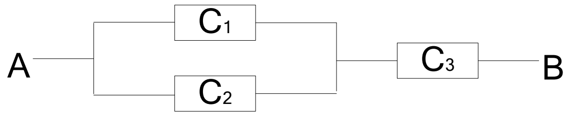


FIGURE 1a The system reliability block diagram.

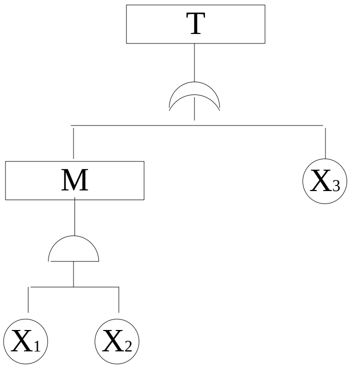


FIGURE lb The system reliability FT

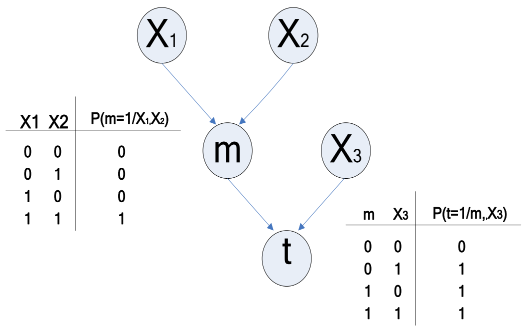


FIGURE lc System reliability BN modeling

Figure 1 two state system reliability modelling based on BN:

$$\begin{aligned} &P(t=1) = \sum_{X_1, X_2, X_3, m} P(X_1, X_2, X_3, m, t) = \\ &\sum_{X_3, m} P(t = 1/m, X_3) \sum_{X_1, X_2} P(m / X_1, X_2) P(X_1) P(X_2) = \\ &\sum_{X_3, m} P(t = 1/m, X_3) P(X_1 = 1) P(X_2 = 1) = \\ &1 - (1 - P(X_1 = 1) P(X_2 = 1)) P(X_3 = 0). \end{aligned} \quad (3)$$

So the top event probability is that:

$$P(t = 1) = 1 - (1 - 0.0008) \times (1 - 0.01) = 0.010792.$$

The system reliability is:

$$R_s = 1 - P(t) = 1 - 0.010792 = 0.98920.$$

4 Multi-state system reliability model based on BN

Multi-state system is divided into discrete multi-state system and continuous multi-state systems. The systems and components state are limited or discrete, which is known as discrete multi-state system. For example, diode has open circuit, short circuit and working, which is three states system and if a system has the following four states:

1) The system working fine (perfect condition).
2) The system in degradation working state.
3) The system is not working because of the fault.
4) The system is not working because of maintenance program. We can use 0,1,2,3 expressing the four states system. In this paper, we study this kind of discrete multi-state system.

Multi-state reliability BN model is described by three different systems in the following. If the component has open circuit and short circuit, two failure modes, such systems also have two kinds of failure modes. In the same situation, the component has open circuit, short circuit and normal state, three states, the system composed of such components has the same three states.

In this paper we use this kind of three states as the example to study the multi-state reliability BN model.

4.1 PARALLEL SYSTEM OF TWO THREE-STATE COMPONENTS

The parallel system reliability block diagram is shown in Figure 2a. A open circuit component will not cause the system failure, and a short circuit component will cause the system failure. Figures 2b and 2c are system reliability block diagram in the two failure modes.

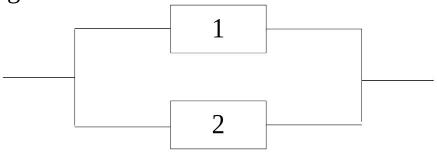


FIGURE 2a The system reliability block diagram

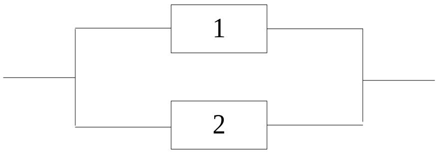


FIGURE 2b Open circuit failure mode

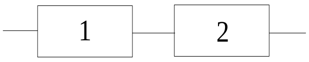


FIGURE 2c Short circuit failure mode

Figure 2 Parallel system of two three-state components.

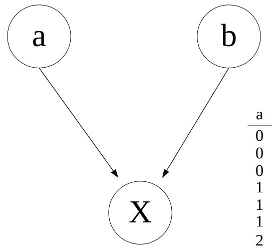
According to literature [24]:

$$Q_0 = q_{01}q_{02}, Q_s = 1-(1-q_{s1}(1-q_{s2})) . \tag{4}$$

In the type, Q_0 is open circuit system failure probability, Q_s is short circuit system failure probability, q_{oi} is open circuit failure probability of component i; q_{si} is short circuit failure probability of component i; R_s is the working probability of component. Then the working probability of the system is that:

$$R_s = 1-Q_0-Q_s = (1-q_{s1})(1-q_{s2})-q_{01}q_{02} . \tag{5}$$

If applying the method, when the component number increasing, not only the minimal path sets and the minimal cut sets is difficult to be got, but no doubt the above formula will become more complex computation. Comparing the above method, we use BN reliability model to solve the problem in the following. It is shown in Figure 3. We use 0, l, 2 to represent system and component open circuit, short circuit state and normal state respectively. P is system or component state probability; node a, b represent two state component; X is the system state.



a	b	P(X/a,b)
0	1	1
0	2	2
0	0	0
1	1	1
1	2	2
1	0	1
2	1	2
2	2	2
2	0	2

FIGURE 3 BN reliability model of parallel system

$$P(X) = \sum_{a,b} p(a,b,X) = \sum_{a} P(a) \sum_{b} [P(X/b)P(b)] = P(a)P(b). \tag{6}$$

$P(a)$ = 1 or P(b) = 1, $P(X)$ = 1;

$P(a)$ = 0 or P(b) = 0, $P(X)$ = 0;

$P(a) \neq 1$, $P(b)$=2 or $P(a)$ = 2 $P(b) \neq 1$, $P(X)$=2.

4.2 SERIES SYSTEM OF TWO THREE-STATE COMPONENTS

The series system reliability block diagram is shown in Figure 4a. A open circuit component will cause the system failure, and a short circuit component will not cause the system failure. Figures 4b and 4c are system reliability block diagram in the two failure modes.

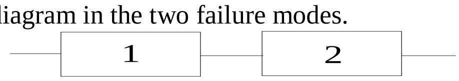


FIGURE 4a The system reliability block diagram

FIGURE 4b Open circuit failure mode

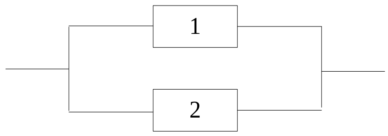


FIGURE 4c Short circuit failure mode

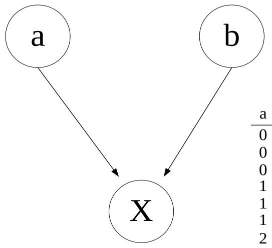
Figure 4 Series system of two three-state components.

According to literature [24]:

$$Q_0 = 1-(1-q_{01}(1-q_{02})), Q_s = q_{s1}q_{s2} , \tag{7}$$

$$R_s = 1-Q_0-Q_s = (1-q_{01})(1-q_{02})-q_{s1}q_{s2} . \tag{8}$$

In the same way, we use BN reliability model to solve the problem in the following. It is shown in Figure 5. We use the CPD to analyse the node X.



a	b	P(X/a,b)
0	1	0
0	2	0
0	0	0
1	1	1
1	2	1
1	0	0
2	1	1
2	2	2
2	0	0

FIGURE 5 BN reliability model of series system

$$P(X)=\sum_{a,b}p(a,b,X)=\sum_{a}P(a)\sum_{b}[P(X/b)P(b)]=P(a)P(b), \quad (9)$$

$P(a)$ = 0 or P(b) = 0, $P(X)$ = 0;

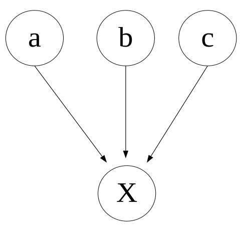
$P(a)$ = 1 or P(b) = 1, $P(X)$ = 1;

$P(a) \neq 0$, $P(b)$=2 or $P(a)$ = 2 $P(b) \neq 0$, $P(X)$=2.

Figures 3 and 5 show, for the same number of multi-state components of the series system and parallel system, the reliability model based on BN is consistent in form, and only the CPD is different, so the multi-state system can be expressed by adjusting the CPD.

4.3 *k*-OUT-OF-*n* SYSTEM OF THREE-STATES COMPONENTS

k-out-of-*n* system can work if at least k components are working. In Figure 4, two-out-of-three of three-state component BN model is established. We use 0, l, 2 to represent system and component state probability respectively. Nodes *a*, *b*, *c* represent the three basic event; node *X* represents system.



a	b	c	P(X/a,b,c)
0	0	0	0
0	0	1	0
0	0	2	0
0	1	0	0
0	1	1	1
0	2	0	0
0	2	2	2
1	0	0	0
1	0	1	1
1	0	2	1
1	1	0	1
1	2	0	1
1	1	1	1
1	2	2	2
2	0	0	0
2	0	1	1
2	0	2	2
2	1	0	1
2	2	0	2
2	1	1	1
2	2	2	2

FIGURE 6 BN reliability model of two-out-of-three system

The main establishment steps of multi-state system reliability model based on BN network is as following:

1) Determine the BN network node. The network root node represents basic events, the leaf node represents the system.

2) Determine the multiple states of discrete systems and components

3) Give the state probability of each component, which is usually given by actual test data.

4) Describe various components state relationship with the CPD; express the associated node state; establish of the BN model of system reliability.

Through the above three systems, we can know that multi-state system reliability model based on BN network has better visual image, and the state is expressed more clearly. Although as the number of system components increasing, CPD expression is more complex, but CPD of BN network is simple, regular, and suitable for programming.

5 Complex multi-state system reliability analyses

Figure 7 is a radar system, which consists of eight sub-system components, antennas X_1, receiver X_2, transmitter X_3, actuators X_4, display screen consisted of a color display instrument X_5, and two series of black and white display instrument $X_6$and X_7 made in parallel, signal processor X_8, data processor X_9, parallel data bus X_{10} and X_{11}, other subsystems can be considered as basic components. In which signal processor and data processor has a co-processing functions, in order to improve reliability, assuming that the two subsystems with memory and compensation, they can work in reduction success.

Supposed X_1, X_3, X_4, X_5, X_6 having all three kinds of states: 0 (failed), 1 (reduction success) and 2 (success); $X_2, X_7, X_8, X_9, X_{10}, X_{11}$ having only two kinds of states: 0 (failed) and 1 (success). In the following, we establish the multi-state fault tree and BN of this system separately. Through analysis, we can see that the method of BN has more analytical modelling than the traditional multi-state fault tree analysis methods.

5.1 ANALYSIS

Figure 8 shows the corresponding radar system multi-state fault tree, which gives the state space of middle events and top event.

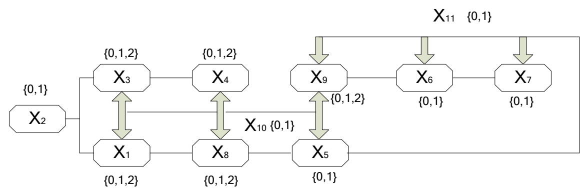


FIGURE 7 A radar system

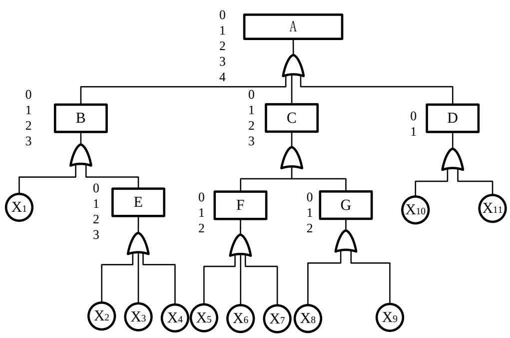


FIGURE 8 Multi-state system fault tree

The multi-state logic operator of middle events and top event is as follows:

TABLE 1 Logic operator

A	**(B,C,D)**
0	All vector including 0
1	111
2	121,131,211,221,311
3	231,321
4	331
B	**(X1,E)**
0	00,01,02,10,20,30
1	11
2	12,21,22,31
3	32
C	**(F,G)**
0	00,01,02,10,20
1	11
2	12,21
3	22
D	**(X10,X11)**
0	00
1	01,10,11
E	**(X2,X3,X4)**
0	All vector including 0
1	111
2	112,121
3	122
F	**(X5,X6,X7)**
0	000,010,001
1	011
2	100,101,110,111
G	**(X8,X9)**
0	00,01,10
1	02,11,12,20,21
2	22

All operator are translated into CPD. The CPD E, A are as follows:

P(E=0/X2=0)=1
P(E=0/X3=0)=1
P(E=0/X4=0)=1
P(E=1/X2=1, X3=1, X4=1)=1
P(E=2/X2=1, X3=1, X4=2)=1
P(E=2/X2=1, X3=2, X4=1)=1
P(E=3/X2=1, X3=2, X4=2)=1
P(A=0/B=0)=1
P(A=0/C=0)=1
P(A=0/D=0)=1
P(A=1/B=1, C=1, D=1)=1
P(A=3/B=2, C=3, D=1)=1
P(A=3/B=3, C=2, D=1)=1
P(A=4/B=3, C=3, D=1)=1
P(A=0/else)=1

Figure 9 shows the BN reliability model of this radar system. Table 2 gives the conditional probability of each root node.

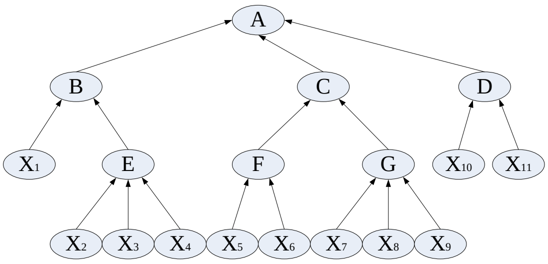


FIGURE 9 Radar system BN reliability model

TABLE 2 Root node conditional probability

	X1	**X3**	**X4**	**X5**	**X6**
0	0.008	0.034	0.001	0.008	0.002
1	0.042	0.058	0.033	0.09	0.04
2	0.95	0.908	0.966	0.902	0.958

	X2	**X7**	**X8**	**X9**	**X10**	**X11**
0	0.025	0.0017	0.0015	0.0015	0.021	0.021
1	0.975	0.9983	0.9985	0.9985	0.979	0.979

5.2 RESULTS

5.2.1 Top event probability

In the traditional multi-state fault tree analysis, top event probability need to calculate the system minimal cut sets and minimal path sets. Based on BN, we can calculate each node probability directly, avoiding non-payment computing. In this radar system, the top event A probability formula and procedure is as following:

$$P(A=i)=\sum_{B,C,D\ldots X_{11}} P(B,C,D,\ldots,X_{10,}X_{11},A=i) \quad . \tag{10}$$

$$B,C,E\in\{0,1,2,3\},D,F,G,X_1,X_3,X_4,X_5,X_6\in\{0,1,2\}$$
$$X_2,X_7,X_8,X_9,X_{10},X_{11}\in\{0,1\},i=0,1,2,3,4$$

TABLE 3 A probability in each state

A	**0**	**1**	**2**	**3**	**4**
probability	0.0675169	0.00000002	0.0167952	0.215148	0.70054

Bayes Net Toolbox (BNT) has the corresponding procedure to solve the problem to simplified the computing. The BNT procedure is:

```
N=18;
dag=zero(N,N)
X1=1; X2=2; X3=3; X4=4; X5=5; X6=6; X7=7; X8=8;
X9=9; X10=10; X11=11; E=12; F=13; G=14; B=15;
C=16; D=17; A=18
dag(X2,E)=1;
dag(X3,E)=1;
dag(X4,E)=1;
dag(X5,F)=1;
dag(X6,F)=1;
dag(X7,G)=1;
dag(X8,G)=1;
dag(X9,G)=1;
dag(X10,D)=1;
dag(X11,D)=1;
dag(X1,B)=1;
dag(E,B)=1;
dag(F,C)=1;
dag(G,C)=1;
dag(B,A)=1;
dag(C,A)=1;
dag(D,A)=1;
discrete_nodes=1:N;
node_size=2 *ones(1,N);
bnet=mk_bnet(dag, node_sizes, 'discrete', discrete_nodes);
bnet.CPD[X1]= tabular_CPD(bnet, X1, [0.008 0.042
0.95]);
```

bnet.CPD[X2]= tabular_CPD(bnet, X2, [0.025 0.975]);
bnet.CPD[X3]= tabular_CPD(bnet, X3, [0.001 0.033 0.966]);
bnet.CPD[X4]= tabular_CPD(bnet, X4, [0.008 0.042 0.95]);
bnet.CPD[X5]= tabular_CPD(bnet, X5, [0.008 0.09 0.902]);
bnet.CPD[X6]= tabular_CPD(bnet, X6, [0.002 0.04 0.958]);
bnet.CPD[X7]= tabular_CPD(bnet, X7, [0.0017 0.9983]);
bnet.CPD[X8]= tabular_CPD(bnet, X8, [0.0015 0.9985]);
bnet.CPD[X9]= tabular_CPD(bnet, X9, [0.0015 0.9985]);
bnet.CPD[X10]= tabular_CPD(bnet, X10, [0.021 0.979]);
bnet.CPD[X11]= tabular_CPD(bnet, X11, [0.021 0.979]);
bnet.CPD[X11]= tabular_CPD(bnet, X11, [0.021 0.979]);
bnet.CPD[B]= tabular_CPD(bnet, B, [1 0 0 0 0 1 1 1]);
bnet.CPD[C]= tabular_CPD(bnet, C, [1 0 0 0 0 1 1 1]);
bnet.CPD[D]= tabular_CPD(bnet, D, [1 0 0 0 0 1 1 1]);
bnet.CPD[E]= tabular_CPD(bnet, E, [1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1]);
bnet.CPD[F]= tabular_CPD(bnet, F, [1 0 0 0 0 1 1 1]);
bnet.CPD[G]= tabular_CPD(bnet, G, [1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1]);
bnet.CPD[A]= tabular_CPD(bnet, A, [1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1]);
engine=jtree_inf_engine(bnet);
evidence=cell(1,N);
evidence{A}=3;
[engine, lolik]=enter_evidence(engine, evidence);
evid.T
ans=0.215148

5.2.2 The importance

In the traditional multi-state fault tree analysis, the importance of components E_i need to get all the quality implication set, and then calculate the importance index E_i. However based on BN, the importance of components E_i can be directly calculated by the conditional probability components. Here we take the RAW (Risk Achievement Worth) as an example, and other type of importance can be calculated according to their definition.

Supposed the state space of system *TE* is (0,1,…,*M*), the state space of component E_i is (0,1,…,M_i), the RAW importance in state *L* can be calculated by type.

$$R_l(i,j) = \frac{1}{M_i+1}\sum_{j=0}^{M_i}\frac{P(TE=l\setminus E_i=j)}{P(TE=l)}.$$

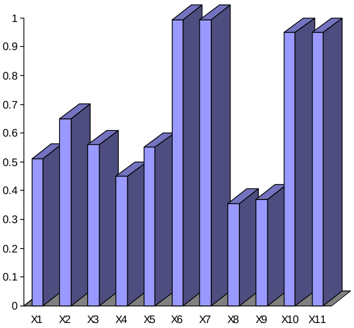


FIGURE 10 RAW importance in state 4

Figure 10 shows the various component RAW importance in state 4. We can be seen from Figure 9, components X_6, X_7 has the greatest importance, while X_8 has the smallest importance. So we can improve the system reliability based on the importance analysis.

5.2.3 Posterior probability

In addition, BN can get more rich information, such as the posterior probability. Supposed system failure, in order to diagnosing the fault and improving the system reliability, we need to compute all nodes posterior probability, while BN gives the fix computing and procedure, which need not compute all node combination, and get the best result.

Supposed system fault at the moment *T*, compute the posterior probability of component X_5.

$$P(X_5/A) = \frac{P(A=1,X_5=1)}{P(A)} = \frac{\sum_{X_1,X_2\cdots X_{11}} P(A=1,X_1,X_2\cdots X_{11}=1)}{P(A)}.$$

BNT procedure is as following:

evidence=cell(1,N);
evidence[A]=1;
[engine, loglik]=enter_evidence(engine, evidence);
marg=marginal_nodes(engine, X5 =1);
marg.T
ans=0.4235

Figure 11 shows probability distribution of each node changing when subsystem *C* transferring from state 3 into state 2. Obviously, we can get the information that it is probably that the state changing of component X_5 and X_6 lead to the subsystem *C* changing. Therefore if monitoring subsystem *C* changing from the intact state 3 to state 2, it should firstly investigate X_5 and X_6 in order to improve the system reliability.

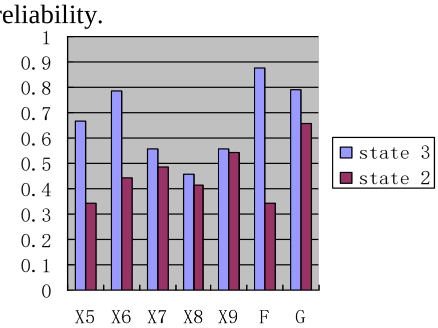


FIGURE 11 Each node probability distribution changing

6 Conclusions

1) According to the BN's two-way uncertainty logical reasoning ability and parallel computing characteristics, we study two state and multi-state system reliability modelling and assessment based on BN.
2) Multi-state system reliability modelling based on BN has the features of the good structure and hierarchy, and

the expression of CPD makes multi-state relationship between components and systems more simple and intuitive. Such as the same series and parallel systems with the same components, because BN form is the same, we can reflect the different system just by adjusting the CPD.

3) During computing the model's reliability, we do not find the system minimal cut sets or minimal path sets, so as to simplify the calculation and greatly enhance the computing accuracy and efficiency. The calculated results of the examples of show the effectiveness and advantage of multi-state system reliability modelling and evaluation based on BN.

4) At last, by means of an example of multi-state radar system, we give the detailed multi-state system reliability analysis process based on BN. The topology of the BN is constructed; the conditional probability distributions and prior distributions are obtained according to multi-state logic operators. Analysis is performed on BN to obtain the probability of top event, importance measures of components and posterior probability and the corresponding formula and procedure is given. Through analysis, we can see that the method of BN has more analytical modelling than the traditional multi-state fault tree analysis methods. This paper not only proves the effectiveness of assessment of multi-state system reliability based on BN, but contributes to good help of complex system reliability, safety analysis.

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Authors

Xiaonan Zhang, born in 1982, Dandong of Liaoling Province, China

Current position, grades: Lecturer at the PLA university of science and technology is engaged in the scientific research work of the Chinese people's liberation army.
University studies: PhD at PLA university of science and military equipment.
Scientific interest: weapons and equipment development, reliability engineering and design.
Publications: More than 20 dissertations.

Haiyong Lu, born in 1974, Haian of Jiangsu Province, China

Current position, grades: Senior engineer of the institute of China electronics technology group.
University studies: Master's at Zhengzhou university in 2002. Professional mechanical and electrical integration.
Scientific interest: Military command and control software.
Experience: Military electronic scientific research work, command and control software in system integration, satellite communications, system reliability, high theoretical level and rich experience in engineering practice, many national and military level scientific research projects.