The approach of fixed asset management based on the shortest path

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Abstract

We often meet with shortest path problem in National Undergraduate Mathematical Contest and practical life. The definition of shortest path problem is introduced, Dijkstra algorithm and 0-1 Programming Method to solve the shortest path problem are given. A practical problem is given and is calculated by these two methods.

Keywords: mathematical modelling, shortest path problem, Dijkstra algorithm, 0-1 Programming

1 Introduction

The shortest path is a very important problem in the graph theory. Many practical problems can be translated into the shortest path problem or into the sub-problem of it, so it is often met with in National Undergraduate Mathematical Contest in Modelling. The Shortest Path Problems are usually used in GISs, [1] and in facility location problems, [2] and in the project design of the laying the pipeline system. How to look for shortest path is the key to solve the intelligence traffic [3]. The Dijkstra algorithm is widely considered to be the excellent algorithm to solve the shortest path in graph theory [4, 5]. And, Moto, et al proposed the method that how to find the shortest path in a certain period of time based on The Dijkstra algorithm [6]. On this basis, someone try to find the method for the extend shortest path [7]. The equipment update timing selection can be generalized into the Shortest Path Problem, as a consequence, these have a practical significance to get hold the method to solve this kind of problem.

2 The Shortest Path Problem and it's solving method

2.1 DEFINITION

The shortest path problem is that to find a path from v_s to v_t in the weighed direct graph (the weight can be the length of the path, or the cost depending on require of specific questions), and the path that all the weighting sum of the arc is minimum number is called be shortest path from v_s to v_t, the weighting sum of the arc is called the distance from v_s to v_t.

2.2 THE SOLVING DIJKSTRA ALGORITHM ON SHORTEST PATH [8]

The Dijkstra algorithm applies to solve the shortest path problem in the condition that the weights w_{ij} of all arc (v_i, v_j) are bigger than 0, so the Dijkstra algorithm can be called double labelling method also. Double labelling method is that assign the point v_j two label (l_j, k_j), the first label l_j mean the length of shortest path and the second k_j mean a subscript of adjacent points front v_j on shortest path from v_s to v_j, then we can find the shortest path from v_s to v_j and the shortest distance from v_s to v_j.

The following is the concrete steps of the Dijkstra algorithm. First, to assign the start point v_1 as label $(0, s)$ that mean the distance is 0 from v_1 to v_1. Second, to suppose a set of labelled points I and a set of unlabelled points J, a set of arcs $\{(v_i, v_j) | v_i \in I, v_j \in J\}$, the arcs in this set are the arcs from the labelled points to the unlabelled points. Last, the calculations are finished if the arc set is empty, the shortest length is l_t if v_t is labelled (l_t, k_t) and the shortest path from v_1 to v_t can get by trace-back from k_t to v_1, the shortest path is non-existent if v_t are never labelled. If the arc set is nonempty, we can work out $s_{ij} = l_i + w_{ij}$ corresponding to every arc (v_i, v_j). There should be the arc that have minimum value in all s_{ij} and this arc is supposed as (v_c, v_d), the ending point v_d of this arc as (s_{cd}, c). Return to the second step. If

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there are many arcs that have the minimum value s_{ij} in third step, we can choose any one to label the end points of these arcs, or to label on every one also.

2.3 THE SOLVING SHORTEST PATH PROBLEM ON THE 0-1 PROGRAMMING THEORY [9]

Suppose v_1 as start point and v_n as end point. Introduce 0-1 decision variable x_{ij},

$$x_{ij}=\begin{cases}1, & \text{if the arc}(v_i,v_j)\text{on the shortest path}\\ 0, & \text{if the arc}(v_i,v_j)\text{not on the shortest path}\end{cases}.$$

The arc of all ones that start from point $v_i(1<i<n)$ must be on shortest path if $\sum_{j=1}^{n}x_{ij}=1$ for any vertex $v_i(1<i<n)$, that is to say, this vertex must be on the shortest path, and the arcs from others vertex to this must be on the shortest path, so $\sum_{j=1}^{n}x_{ji}=1$, the vertex $v_i(1<i<n)$ is not on the shortest path if $\sum_{j=1}^{n}x_{ij}=0$, so there will be $\sum_{j=1}^{n}x_{ji}=0$; Combining the above two cases, we can get $\sum_{j=1}^{n}x_{ij}=\sum_{j=1}^{n}x_{ji},1<i<n$. There must be $\sum_{j=1}^{n}x_{1j}=1$ for start point v_1 and there must be $\sum_{j=1}^{n}x_{jn}=1$ for end point v_n. The value of the objective function that sum up weight of every arcs on shortest path is minimum, so 0-1 Programming Model that solve the shortest path problem is as: objective function: $\min z=\sum_{(v_i,v_j)\in E}w_{ij}x_{ij}$ (E is a set of all arc in chart)

$$s.t.\begin{cases}\sum_{(v_i,v_j)\in E}x_{ij}=\sum_{(v_i,v_j)\in E}x_{ji},1<i<n\\ \sum_{(v_1,v_j)\in E}x_{1j}=1,\ \sum_{(v_j,v_n)\in E}x_{jn}=1\\ x_{ij}=0 or 1\end{cases}$$

3 The application of shortest path problem

3.1 EQUIPMENT REPLACEMENT PROBLEM

There is a machine that can work for 4 years continuously, or can be sold at the end of year and buying a new one instead. As following, we have known the price of new machine at beginning of the year and the sold price of each machine that have different enlistment age. The operation costs and maintenance costs of the new machine is 3000 Yuan; and the cost of operation and maintenance of the machine that use 1 to 3 years in each year are 8000yuan ,15000yuan, 20000yuan respectively. How make sure the machine optimal update strategy to attain a least cost that sum up purchase and replacement and maintenance within four years?

TABLE 1 The sold price of the machine at every year

Year j	1	2	3	4
Purchase price at beginning year	2.5	2.6	2.8	3.1
reduced price at end year j	2.0	1.6	1.3	1.1

3.2 TO BE TRANSLATED INTO THE SHORTEST PATH PROBLEM

Equipment replacement can be translated into the shortest path problem, as follow figure 1. The point v_i means that "we purchase a new machine at beginning of the year i", the point v_5 means the end of the fourth year. We will draw arcs from v_i to $v_{i+1},\cdots,v_5$ respectively, the arc (v_i,v_j) means purchase new machine at beginning year i, has been used until the beginning of year j, that is the end of year $j-1$. The weight of the arc (v_i,v_j) is total cost including the purchase cost and the maintenance cost from beginning of year i to the end of year $j-1$ and to subtract the residual value of the equipment at the end of year $j-1$. Example, the weight of arc (v_1,v_5) is that the purchase cost 25000 Yuan at first year plus maintenance cost from beginning of first year to the end of fourth year 3000+8000+15000+20000=46000.

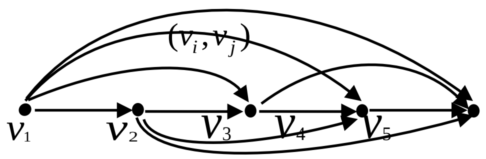

FIGURE 1 Reducing practical problem to Shortest Path Problem

TABLE 2 Weight w_{ij} of all arc (v_i,v_j) /wan Yuan

w_{ij} / j / i	1	2	3	4	5
1	-	0.8	2	3.8	6
2	-	-	1.3	2.4	4.1
3	-	-	-	1.8	2.8
4	-	-	-	-	2.3
5	-	-	-	-	-

Moreover, subtract deduced cost11000 Yuan at end of fourth year, got 60000 Yuan. The weights w_{ij} of all arcs (v_i,v_j) have been shown in tab.2 as following:

So, to find a shortest path from v_1 to v_5, we can attain the optimal equipment replacement strategy, which is the least cost summing up the purchase and the replacement and the maintenance within four years .

3.3 USING DIJKSTRA ALGORITHM TO SOLVE THE SHORTEST PATH PROBLEM

The Dijkstra algorithm can be used to solve the shortest path as follow:

(1) The point v_1 is labelled as $(0,s)$, there is $I=\{v_1\}$, $J=\{v_2,v_3,v_4,v_5\}$, the set of arc is $\{(v_i,v_j)|v_i \in I, v_j \in J\} = \{(v_1,v_2),(v_1,v_3),(v_1,v_4),(v_1,v_5)\}$, and $s_{12}=l_1+w_{12}=0+0.8=0.8$, $s_{13}=l_1+w_{13}=0+2=2$. Similarly, $s_{14}=3.8$, $s_{15}=6$, $\min(s_{12},s_{13},s_{14},s_{15})=s_{12}=0.8$, the end point v_2 of the arc (v_1,v_2) is labelled as $(0.8,1)$.

(2) There is $I=\{v_1,v_2\}$, $J=\{v_3,v_4,v_5\}$, set of arc is $\{(v_1,v_3),(v_1,v_4),(v_1,v_5),(v_2,v_3),(v_2,v_4),(v_2,v_5)\}$, and

$s_{23}=l_2+w_{23}=0.8+1.3=2.1$,

$s_{24}=l_2+w_{24}=0.8+2.4=3.2$,

$s_{25}=l_2+w_{25}=0.8+4.1=4.9$,

$\min(s_{13},s_{14},s_{15},s_{23},s_{24},s_{25})=s_{13}=2$, the end point v_3 of the arc (v_1,v_3) are labelled $(2,1)$

(3) there is have $I=\{v_1,v_2,v_3\}$, $J=\{v_4,v_5\}$, set of arc is $\{(v_1,v_4),(v_1,v_5),(v_2,v_4),(v_2,v_5)(v_3,v_4),(v_3,v_5)\}$, and $s_{34}=l_3+w_{34}=2+1.8=3.8$, $s_{35}=l_3+w_{35}=2+2.8=4.8$, $\min(s_{14},s_{15},s_{24},s_{25},s_{34},s_{35})=s_{24}=3.2$, the end point v_4 of the arc (v_2,v_4) are labelled $(3.2,2)$.

(4) there is have $I=\{v_1,v_2,v_3,v_4\}$, $J=\{v_5\}$, set of arc is $\{(v_1,v_4),(v_1,v_5),(v_2,v_4),(v_2,v_5)(v_3,v_4),(v_3,v_5)\}$, and $s_{45}=l_4+w_{45}=3.2+2.3=5.5$,

$\min(s_{15},s_{25},s_{35},s_{45})=s_{35}=4.8$, the end point v_5 of the arc (v_3,v_5) are labelled $(4.8,3)$.

So, the length of shortest path is 4.8 from v_1 to v_5 and the shortest path is $v_1 \to v_3 \to v_5$, that is the cost of this layout that we will purchase a new machine at the beginning first year and deal with it at the end of second year along with purchase a new machine at the beginning third year and deal with it at the end of fourth year is 48000 yuan.

3.4 0-1 PROGRAMMING AND LINGO PROGRAM

Introduce 0-1 decision variable x_{ij} [10],

$$x_{ij}=\begin{cases}1, & \text{if the arc}(v_i,v_j)\text{on the shortest path}\\ 0, & \text{if the arc}(v_i,v_j)\text{not on the shortest path}\end{cases}$$

Therefore, 0-1 Programming model to solve the shortest path problem is as:

Objective function $\min z=\sum_{(v_i,v_j)\in E} w_{ij}x_{ij}$ (E is a set of all arc in chart)

$$s.t.\begin{cases}\sum_{(v_i,v_j)\in E} x_{ij} = \sum_{(v_i,v_j)\in E} x_{ji}, 1<i<n\\ \sum_{(v_1,v_j)\in E} x_{1j}=1,\ \sum_{(v_j,v_n)\in E} x_{jn}=1\\ x_{ij}=0 \text{ or } 1\end{cases}$$

The answer can be found by using LINGO Program as following:

```
model:
sets:years/v1,v2,v3,v4,v5/;
roads(years,years)/
v1,v2 v1,v3 v1,v4 v1,v5 v2,v3 v2,v4 v2,v5 v3,v4
v3,v5 v4,v5/:W,X;
endsets
data:
w=0.8 2 3.8 6 1.3 2.4 4.1 1.8 2.8 2.3;
enddata
N=@SIZE(YEARS);
MIN=@SUM(roads:W*X);
@for(years(i)|i #GT# 1 #AND# i #LT# N:
@SUM(roads(i,j):X(i,j))=@SUM(roads(j,i):X(j,i)));
@SUM(roads(i,j)|i #EQ# 1:X(i,j))=1;
@SUM(roads(i,j)|j #EQ# N:X(i,j))=1;
end
```

The answer made by using computer is the same with using Dijkstra algorithm; the computer is more efficient than Dijkstra algorithm. Its drawback is ask for a shortest path from start point to end point are appointed and we must change procedure and recalculate if change start point.

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